UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF LAND MANAGEMENT

EXAMPLES OF GOOD PRACTICE

OBSERVATIONS

FOR

TIME, LATITUDE, AND AZIMUTH

WITH

REDUCTIONS AND RELATED CALCULATIONS

Washington, D. C.
Reprint of 1957

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF LAND MANAGEMENT

Foreword

This is a reprint of the publication first issued in April, 1946, by the former General Land Office, a predecessor of the Bureau of Land Management. It is reissued to meet the needs of the field cadastral surveying personnel who have found these Examples of Good Practice valuable for reference and as a guide in the making of similar observations.

The original publication resulted from a recommendation of the Board on Technical Procedures of the General Land Office to the effect that special examples of good practice in particular phases of the work be assembled and distributed for the benefit of the entire cadastral surveying service for reference and use in similar cases. There are included herein examples of basic methods for determination of time, latitude, and azimuth by observation of the sun, Polaris, and selected bright stars within the equatorial belt.

The recorded observations were made by cadastral engineers in the execution of their regular field assignments in the survey and resurvey of the public lands. The examples illustrate the approved methods described in the Manual of Instructions for the Survey of the Public Lands of the United States, edition of 1947. The basic astronomical data are as supplied in the Ephemeris, published annually by this Bureau.

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All observations made with the General
Land Office one-minute solar transit

Example of daylight altitude observation of a star for time and azimuth:

Place: Washington, D. C. Latitude: 38° 53' 30" N. Longitude: 77° 02' W.

Date: July 28, 1944. Observer: J. C. Thoma Instrument: Buff No. 23461
Recorder: O. B. Walsh
Time : A. D. Kidder

Sun's decl., Gr. app. noon	18°57'28.4" N.
Red. to long. 77°02'W. or 5^h08^m	
" " time of obsn. p.m. 4 30 = 9.63 x 34.8" (335")	5 35 (S.)
Refraction in polar distance	1 02 (N.)
The sun's app. decl.	18°52'55" N.

At 4^h30^m p.m., app. t., I set the arcs of the solar unit, lat. 38°53'30" N., and decl. 18°53'N., and orient to the meridian, setting a reference mark South.

App. t. of sunset 7^h05^m p.m.
Equa. of time (add) 6

Sunset, l.m.t. 7^h11^m p.m.

Star: 25/49 0.9
α Aquilae (Altair)
δ = +8°43.4'

Latitude: 38°53'30"N.

Gr.m.t. star's transit, July 19:		$11^h57.5^m$ p.m.
Red. to " 28:	-	35.4
" " long. 77°02'W. :	-	0.8
Star's transit, l.m.t. " 28:	=	$11^h21.3^m$ p.m.
Anticipated l.m.t. of obsn. :		6 40 "
Hour angle SE. (70°15')		4^h41^m

Watch: O.K.: l.m.t.
: slow of E.W.T.: 1^h08^m

$$\sin h = \cos t \cos \varphi \cos \delta + \sin \varphi \sin \delta \qquad \cos A = \frac{\sin \delta}{\cos \varphi \cos h} - \tan \varphi \tan h$$

	cos	sin	
t =	.3379		
φ =	.7783	.6278	
δ =	.9884	.1516	
	.2599	.0952	(Products)
		.2599	
sin h =		.3551	(Sum)
h =	20°48'		

cos	sin	tan	
.7783		.8067	
	.1516		
.9348		.3799	
.7276		.3065	(Products)
	.2084	.2084	(Fraction)
cos A =		.0981	(Diff. -)

A = S.84°22'E.

See page 2 for observation.

With the above settings, at 6^h40^m p.m., l.m.t., I find the star in good position and proceed with 6 observations, the 1st as follows:

Watch time of obsn.: $6^h45^m20^s$ p.m.
Horizontal angle from star to reference mark: 83°36'00"
Observed vertical angle: v = 21°44'
Refraction: r = – 2'24"

True vertical angle: h = 21°41'36"

$$\cos t = \frac{\sin h}{\cos \varphi \cos \delta} - \tan \varphi \tan \delta \qquad \cos A = \frac{\sin \delta}{\cos \varphi \cos h} - \tan \varphi \tan h$$

	cos	sin	tan	
h =		.36964		
φ =	.77833		.80666	
δ =	.98844		.15342	
	.76933		.12376	(Products)
		.48050		(Fraction)
		.12376		
cos t =		.35674		(Diff. +)

	cos	sin	tan	
	.92918		.39781	
	.77833		.80666	
		.15166		
	.72321		.32090	(Products)
		.20970	.20970	(Fraction)
cos A =			.11120	(Diff. -)

t = 69°06'
= $4^h36^m24^s$ Sidereal h.a.
– 44 Red. to m.t.h.a.

$4^h35^m40^s$ Mean time h.a.
11 21 18 Star's transit, p.m., l.m.t.

$6^h45^m38^s$ Correct l.m.t. of obsn.
6 45 20 Watch time of obsn.

0^m18^s Watch slow of l.m.t.

A = S.83°37'00"E.
83°36'00" Obsvd. hor. ang.
S. 0°01'00"E. Reference mark.

	Time	Observed Vert. Ang.	Observed Hor. Ang.	Watch slow, l.m.t.	Bearing reference mark
(1)	Reduced bearing of reference mark " watch slow of l.m.t.			18^s	S.0°01'00"E.
2	$6^h51^m00^s$	22°50'30"	82°40'00"	24	S.0°00'30"W.
3	6 57 30	24°05'00"	81°34'00"	20	South
4	7 00 00	24°35'00"	81°08'00"	25	S.0°00'30"W.
5	7 04 54	25°31'00"	80°18'00"	22	S.0°00'30"W.
6	7 08 12	26°09'00"	79°43'00"	23	South
			Mean	22^s	S.0°00'05"W.

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α Pegasi.

Date: Oct. 31, 1944: Instrument: Gurley No. 371653

Observer: Lloyd E. Toland
Recorder and time: Jesse L. Gassman.

Sun's decl., Gr. app. noon	14° 09' 30.7" S.
Red. to long. 104° 28' W. or 6h 58m	
" " time of obsn. p.m. 4 02 11 x 48.55" (534")	8 54 (S)
Refraction in polar distance	2 14 (N)
The sun's app. decl.	14° 16' 11" S.

At 4h 02m p.m., app. t., I set off the arcs of the solar unit, lat. 32° 50' 30" N., and decl. 14° 16' S., and orient to the meridian, setting a reference mark South.

App. t. of sunset	5h 22m p.m.
Equa. of time (sub)	16
Sunset, l.m.t.	5h 06m p.m.

Star: 28/55 a Pegasi 2.6
+ 14° 54.6'

Latitude: 32° 50' 23.212" N.

Gr.m.t, star's transit, Oct.	16:	9h 21.0m p.m.
Red. to "	31:-	59
" " long. 104° 28'W.	:-	1.1
Star's transit, l.m.t. "	31	8h 20.9m p.m.
Anticipated l.m.t, of obsn.		5 21
Hour angle SE. (45°)		3h 00m

Watch: O.K.; M.W.T. by radio.
" : fast l.m.t. 57m 52s

$$\sin h = \cos t \cos \phi \cos d + \sin \phi \sin d \; ; \quad \cos A = \frac{\sin d}{\cos \phi \cos h} - \tan \phi \tan h$$

	cos	sin
t =	.70711	
ϕ =	.84019	.54229
d =	.96633	.25730
	.57410	.13953
		.57410
sin h =		.71363

h = 45° 32'

cos	sin	tan
.84019		.64544
	.25730	
.70049		1.01879
.58854		.65757
	.43718	.43718
cos A =		.22039

A = S. 77° 16' E.

With the above settings, at 5h 13m p.m., l.m.t., I find the star in good position and proceed with 6 direct and 6 reversed readings; recording the mean of the 1st. direct and 6th reversed, 2nd direct and 5th reversed, etc. The first mean as follows:

Watch time of obsn.:	6h 13m 34s p.m.
Horizontal angle from star to reference mark:	78° 17' 30"
Observed vertical angle:	v = 44° 22' 30"
Refraction:	r = 1' 00"
True vertical angle:	h = 44° 21' 30"

$$\cos t = \frac{\sin h}{\cos \phi \cos d} \qquad \cos A = \frac{\sin d}{\cos \phi \cos h} - \tan \phi \tan h$$

	cos	sin	tan
h =		.69915	
ϕ =	.84020		.64544
d =	.96634		.26623
	.81192		.17184
		.86111	
		.17184	
cos t =		.68927	

t = 46° 25' 40"
= 3h 5m 43s Sideral h.a.
\- 1 9 Red. to m.t.h.a.
3h 4m 34s Mean time h.a.
8h 20m 54s Star's transit, p.m., l.m.t.
5h 16m 20s Correct l.m.t. of obsn.
6 13 34 Watch time of obsn.
57m 14s Watch fast of l.m.t.

cos	sin	tan
.71498		.97784
.84020		.64544
	.25727	
.60073		.63114
	.42826	.42826
cos A =		.20288

A = S. 78° 17' 42" E.
78° 17' 30" Obsvd. h. a.
S. 0° 00' 12" E. Ref. mkd.

				Watch fast, l.m.t.	Bearing reference mark
(1)	Reduced bearing of reference mark				S.0°00'12"E.
	" watch fast of l.m.t.			57m14s	
	Time	Observed Vert. Ang.	observed Hor. Ang.		
2	6h13m28s	44°21'30"	78°18'00"	57 13	S.0°00'30"E.
3	6 13 25	44°21'00"	78°18'30"	57 03	S.0°00'20"E.
4	6 13 20	44°20'30"	78°18'30"	57 09	S.0°00'41"E.
5	6 13 18	44°19'30"	78°19'30"	57 12	S.0°00'23"E.
6	6 13 12	44°20'00"	78°19'00"	57 05	S.0°00'32"E.
			Mean	57m09s	S.0°00'26"E.

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B. Orionis.

Date: Nov. 1, 1944. Observer: Lloyd E. Toland Instrument:
Recorder and time: Jesse L. Gassman Gurley No. 371653

Reference mark set South as used in observation on a Pegasi, Oct. 31, 1944.

App. t. of sunrise 6h39m a.m. Gr.m.t. star's transit, Nov. 1: 2h30.8m a.m.
Equa. of time (sub) 16 Red. to long. 104° 28' W. :- 1.1
Sunrise, l.m.t. 6h23m a.m. Star's transit, l.m.t. " 1: 2h29.7m a.m.
Star: 7/11 0.3 Anticipated l.m.t. of obsn. : 6 02 a.m.
β Orionis (Rigel) Hour angle SW. (53°) 3h32m
- 8°15.8'
Watch: fast l.m.t.: 30s
Latitude: 32°50'23"N. " : " M.W.T.:58m22s

$\sin h = \cos t \cos \phi \cos d \overset{(-)}{+} \sin \phi \sin d;\ \cos A = \dfrac{\sin d}{\cos \phi \cos h} \overset{(+)}{-} \tan \phi \tan h$

	cos	sin	cos	sin	tan
t =	.60182				
φ =	.84020	.54228	.84020		.64544
d =	.98962	.14372		.14372	
	.50040	.07794			
	.07794				
sin h =	.42246				
h =	24°59'		.90643		.46595
			.76158		.30074
				.18871	.18871
			cos A =		.48945
			A =	S.60°42'W.	

With the above settings, at 6h02m a.m., l.m.t., I find the star in good position and proceed with 6 direct and 6 reversed readings, with the reversal made at the end of the 6th direct reading, recording the mean of the first direct and 6th reverse, the 2nd direct and 5th reverse readings, etc. The 1st as follows:

Watch time of observation: 7h05m16s a.m.
Horizontal angle from star to reference mark: 61°50'
Observed vertical angle: v = 23°52'
Refraction: r = 2'10"
True vertical angle: h = 23°49'50"

$\cos t = \dfrac{\sin h}{\cos \phi \cos d} \overset{(+)}{-} \tan \phi \tan d \qquad \cos A = \dfrac{\sin d}{\cos \phi \cos h} \overset{(+)}{-} \tan \phi \tan h$

	cos	sin	tan	cos	sin	tan
h =		.40404		.91476		.44169
φ =	.84020		.64544	.84020		.64544
d =	.98962		.14520		.14372	
	.83148		.09372	.76856		.28508
		.48593			.18700	.18700
		.09372				
cos t =		.57965		cos A =		.47208

t = 54°34'30" A = S.61°49'50"W.
= 3h38m18s Sideral h.a. 61°50'00" Obsvd. h. a.
\- 1 09 Red. to m.t.h.a. S. 0°00'10"E. Ref. mark.
3h37m09s Mean time h.a.
2 29 42 Star's transit, a.m., l.m.t.
6h06m51s Correct l.m.t. of obsn.
7 05 16
58m25s Watch fast of l.m.t.

	Time.	Observed Vert. Ang.	observed Hor. Ang.	Watch fast, l.m.t.	Bearing reference mark
(1)	Reduced bearing of reference mark				S.0°00'10"E.
"	watch fast of l.m.t.			58m25s	
2	7h05m17s	23°51'	61°50'30"	58 21	South
3	7 05 20	23°51'	61°51'00"	58 27	S.0°00'30"E.
4	7 05 25	23°50'	61°51'30"	58 24	S.0°00'14"E.
5	7 05 20	23°52'	61°50'30"	58 29	S.0°00'46"E.
6	7 05 22	23°50'	61°52'00"	58 21	S.0°00'44"E.
			Mean	58m24s	S.0°00'24"E.

B. Orionis.

ALTITUDE OBSERVATION OF A STAR FOR AZIMUTH.

Date	Observer: Norman D. Price	Instrument
October 25, 1944	Recorder: George F. Rigby	Buff No.17998

In order to determine a meridian at my station in the SW$\frac{1}{4}$ of NW$\frac{1}{4}$ of sec.25, T. 1 N., R. 1 E., Willamette Mer., Oregon, in lat. 45°32'30"N., and long. 122°39'W., I proceed with the preliminary preparation, and with a series of six altitude observations, three each with the telescope in direct and reversed positions, upon the star (Alpha) Ophiuchi as follows:

Sun's decl. Greenwich app. noon----------------------------------	12° 09' 08.1" S.
Red. to long. 122° 39'W. 8h10m36s	
Red. to time of obsn. p.m. 4 01 24 = 12.2 x 51.66" =(630.3")	10 30.3 (S.)
Refraction in polar distance	3 50 (N.)
The sun's apparent declination	12° 15' 48" S.

At 4h01.4m p.m., app.t., I set the arcs of the solar unit, lat. 45°32$\frac{1}{2}$'N., and decl. 12°16'S., and orient to the meridian, setting a reference mark South.

App.t. of sunset	5h09m p.m.	Gr. m.t. star's transit	Oct. 16: 3h52.2m p.m.
Equa. of t.	- 16m	Red. to	Oct. 25: -35.4
Sunset, l.m.t.	4h53m p.m.	Red. to long. 122°39'W.	- 1.3
		Star's transit, l.m.t.	Oct. 25: 3h15.5m p.m.
Star: 22/44	2.1	Anticipated l.m.t. of obsn.	Oct. 25: 5h55.0m p.m.
(Alpha) Ophiuchi		Hour angle SW. (39°52.5')	2h39.5m
+12°36.2'			
Latitude 45° 32' 30" N.		Watch: correct: l.m.t.	
		Watch: slow of P.W.T.: 1h 10m 36s	

sin h = cos t cos Ø cos d + sin Ø sin d $\cos A = \dfrac{\sin d}{\cos Ø \cos h} - \tan Ø \tan h$

	cos	sin		cos	sin	tan	
t =	.76744						
Ø =	.70039	.71376		.70039		1.01909	
d =	.97591	.21820			.21820		
	.52456	.15574	(Products)				
		.52456					
sin h =		.68030	(Sum)				
h = 42° 52'				.73294	————	.92817	
				.51334		.94589	(Products)
					.42506	.42506	(Fraction)
				cos A =		.52083	(Diff. -)
				A = S. 58° 37' W.			

With the above settings, at 5h55m p.m.,l.m.t., I find the star in good position and proceed with six observations, the first as follows:

Watch time of obsn.			5h 57m 02s p.m.
Horizontal angle from star to reference mark-----------------------			59° 15' 30"
Observed vertical angle:	v = 42° 31'		
Refraction:	r = -1' 03"	Latitude:	Ø = 45° 32' 30" N.
True vertical angle:	h = 42° 29' 57"	Declination:	d = 12° 36' 12" N.

$$\cos A = \frac{\sin d}{\cos Ø \cos h} - \tan Ø \tan h$$

log sin d	= 9.338855		log tan Ø	=	10.008212	
log cos Ø	= 9.845351		log tan h	=	9.962039	
Difference	= 9.493514		Sum	=	9.970251	log of 2nd term
log cos h	= 9.867637		2nd term	=	.93379	Natural (-)
Difference	= 9.625877	log 1st term	1st term	=	.42255	Natural (+)
1st term	= .42255	Natural (-)	cos A	=	.51124	Diff. (-)
			A	=	59° 15' 12"	
		Observed horizontal angle		=	59° 15' 30"	
		Bearing of reference mark		=	S.0° 00' 18"E.	

	Time	Observed Vert.Ang.	Observed Hor.Ang.	Bearing of Reference mark
(1)	5h 57m 02s	42° 31' 00"	59° 15' 30"	S. 0° 00' 18" E.
(2)	5h 58m 20s	42° 18' 00"	59° 38' 00"	S. 0° 00' 26" E.
(3)	5h 59m 29s	42° 07' 30"	59° 56' 00"	S. 0° 00' 26" E.
(4)	6h 02m 48s	41° 38' 00"	60° 46' 00"	S. 0° 00' 48" E.
(5)	6h 04m 13s	41° 25' 00"	61° 07' 00"	S. 0° 00' 12" E.
(6)	6h 05m 39s	41° 12' 30"	61° 28' 00"	S. 0° 00' 39" E.
			Mean-------------	S. 0° 00' 28" E.

Altitude Observation of the Sun for Apparent Time.

Date	Observer	Instrument
September 4, 1944	Norman D. Price	Buff No.17998

At camp, in sec. 5, T. 25 S., R. 3 W., Willamette Meridian, Oregon, in. lat. 43° 25' 30" N., and long. 123° 04' 25" W., to check the reading of my watch which indicates approximate standard time for the 120th meridian, I make an altitude observation of the sun for time, making two observations, one each with the telescope in reversed and direct positions, observing opposite limbs of the sun, as follows:

Mean observed vertical angle-----------28° 18' 30"
Mean watch time of observation--------- 3h 59m 27s p.m.
Watch fast of l.m.t. (theoretical)----- 12m 18s
Mean temperature at time of obsn.------96°Fahr.
Elevation above sea level of station---950 ft.

Telescope	Sun's limb's		Watch time.	Vertical Angle.
Reversed----------	q		3h 58m 00s p.m.	28° 19' 00"
Direct------------		b	4h 00m 54s	28° 18' 00"
Mean-------------------			3h 59m 27s p.m.	28° 18' 30"=v

Refraction (corrected for temperature and elevation above sea level), 107"x.92x.98= minus 1' 36.47"
Sun's parallax------------------------ = plus 7.79"
h = 28° 17' 01"

True vertical angle=h=28°17'01"
Zenith distance =Z=61°42'59"
Sun's declination =D= 6°57'02"N.
Latitude =ϕ=43°25'30"N.
T=hour angle from apparent noon in angular measure.

$$\tan \tfrac{1}{2} T = \sqrt{\frac{\sin\frac{1}{2}(Z+\phi-D)\sin\frac{1}{2}(Z-\phi+D)}{\cos\frac{1}{2}(Z+\phi+D)\cos\frac{1}{2}(Z-\phi-D)}}$$

```
      Z=61°42'59"                                      Z=61°42'59"
      φ=43°25'30"                                      φ=43°25'30"
    Z+φ=105°08'29"                                   Z-φ=18°17'29"
      D=  6°57'02"(+)        ½ values=                 D= 6°57'02"(+)
  Z+φ+D=112°05'31"---56°02'45" 12°37'15"----Z-φ+D=25°14'31"
    Z+φ=105°08'29"                                   Z-φ=18°17'29"
      D=  6°57'02"(+)                                  D= 6°57'02"(+)
  Z+φ-D= 98°11'27"---49°05'43"  5°40'13"----Z-φ-D=11°20'27"
   Log sin ½(Z+φ-D)=              9.878407
    "  sin ½(Z-φ+D)=              9.339448
                                  9.217855
    "  cos ½(Z+φ+D)=9.747047
    "  cos ½(Z-φ-D)=9.997869
                    9.744916      9.744916   57°=3h48m00s
    "  tan² ½T =                  9.472939   11'=     44s
    "  tan  ½T =                  9.736469   20"=     01s
          ½T = 28°35'40"  T = 57°11'20"       T= 3h48m45s
```

Apparent time of observation---------3h48m45s p.m.
Equation of time---------------minus 1 12
Local mean time of observation-------3h47m33s p.m.
Difference in time for long.----add 12 18
120th mer. time of observation-------3h59m51s p.m.
Watch time of observation------------3h59m27s p.m.
Watch slow of 120th mer. time-------- 24s

Altitude Observation of a Star for Time.

Star 22/44 (Alpha) Ophiuchi, Mag. 2.1
+12°36.2'

Date	Observer: Norman D. Price	Instrument
October 25, 1944	Recorder: George F. Rigby	Buff No.17998

At my station, in SW¼ of NW¼ of sec. 25, T. 1 N., R. 1 E. Willamette Meridian, Oregon, in latitude 45° 32' 30" N., and longitude 122° 39' W., to check the reading of my watch which indicates approximate Pacific war time, I make a series of six altitude observations, three each with the telescope in direct and reversed positions, upon the star (Alpha) Ophiuchi, as follows:

Telescope.	Watch time.	Vertical angle.
Direct------------	7h 08m 07s p.m.	42° 31' 00"
Direct------------	7h 09m 25s	42° 18' 00"
Direct------------	7h 10m 34s	42° 07' 30"
Reversed----------	7h 13m 53s	41° 38' 00"
Reversed----------	7h 15m 18s	41° 25' 00"
Reversed----------	7h 16m 44s	41° 12' 30"

Mean temperature at time of obsn.------------50°Fahr.
Barometric reading at time of obsn.----------29.6 ins.

Transit of (Alpha) Ophiuchi, civil date and mean time, is

3h 52m 12s p.m., October 16, 1944
Sub. 35m 24s correction to October 25

3h 16m 48s p.m., transit Greenwich Meridian
Sub. 1m 20s reduction for longitude of this station

3h 15m 28s p.m., l.m.t. transit of (Alpha) Ophiuchi.

1h 10m 36s-additive for correction to Pacific war time.

$$\cos t = \frac{\sin h}{\cos \phi \cos d} - \tan \phi \tan d$$

Formula for derivation of sidereal hour angle "t" in angular measure.

t = sidereal hour angle expressed in angular measure.
h = vertical angle corrected for refraction, (r)
ϕ = latitude of station = 45° 32' 30" N.
d = declination of star = 12° 36' 12" N.

The following reductions are made to obtain the true vertical angles of the above observations:

	1st obsn.	2nd obsn.	3rd obsn.	4th obsn.	5th obsn.	6th obsn.
v =	42°31'00"	42°18'00"	42°07'30"	41°38'00"	41°25'00"	41°12'30"
r =	-1 03	-1 03	-1 04	-1 05	-1 05	-1 06
h =	42°29'57"	42°16'57"	42°06'26"	41°36'55"	41°23'55"	41°11'24"

Solutions of observations by foregoing formula are as follows:

Two operations are common to reduction of all six observations.

Denominator of 1st term

log cos ϕ = 9.845341
log cos d = 9.989407
Sum = 9.834748 = log SD

Anti-log of 2nd term

log tan ϕ = 10.008212
log tan d = 9.349448
Sum = 9.357660
Anti-log = .22786 = N2

Altitude Observation of a Star for Time.

	1st obsn.		2nd obsn.	3rd obsn.
log sin h =	9.829676	log sin h =	9.827877	log sin h = 9.826412
log SD =	9.834748	subtract	9.834748	9.834748
	9.994928		9.993129	9.991664
anti-log =	.98839	1st term	.98430	.98099
N2 =	.22786	subtract	.22786	.22786
cos t =	.76053		.75644	.75313
t =	40°29'20"		40°50'54"	41°08'16"
=	2h41m57s		2h43m24s	2h44m33s
	-26s	sidereal conversion	-27s	-27s
	2h41m31s	mean time hour angle	2h42m57s	2h44m06s
add	3h15m28s	l.m.t.star's transit	3h15m28s	3h15m28s
	5h56m59s	l.m.t. of obsn.	5h58m25s	5h59m34s
add	1h10m36s	corr. to P.W.T.	1h10m36s	1h10m36s
	7h07m35s	P.W.T. of obsn.	7h09m01s	7h10m10s
	7h08m07s	watch t. of obsn.	7h09m25s	7h10m34s
	32s	watch fast of P.W.T.	24s	24s

	4th obsn.		5th obsn.	6th obsn.
log sin h =	9.822250	log sin h =	9.820394	log sin h = 9.818594
log SD =	9.834748	subtract	9.834748	9.834748
	9.987502		9.985646	9.983846
anti-log =	.97163	1st term	.96749	.96349
N2 =	.22786	subtract	.22786	.22786
cos t =	.74377		.73963	.73563
t =	41°56'47"		42°18'00"	42°38'22"
=	2h47m47s		2h49m12s	2h50m33s
	-28s	sidereal conversion	-28s	-28s
	2h47m19s	mean time hour angle	2h48m44s	2h50m05s
add	3h15m28s	l.m.t.star's transit	3h15m28s	3h15m28s
	6h02m47s	l.m.t. of obsn.	6h04m12s	6h05m33s
add	1h10m36s	corr. to P.W.T.	1h10m36s	1h10m36s
	7h13m23s	P.W.T. of obsn.	7h14m48s	7h16m09s
	7h13m53s	watch t. of obsn.	7h15m18s	7h16m44s
	30s	watch fast of P.W.T.	30s	35s

By 1st obsn. watch fast of P.W.T.	32s
By 2nd obsn. watch fast of P.W.T.	24s
By 3rd obsn. watch fast of P.W.T.	24s
By 4th obsn. watch fast of P.W.T.	30s
By 5th obsn. watch fast of P.W.T.	30s
By 6th obsn. watch fast of P.W.T.	35s
Mean watch fast of P.W.T.	29s

Meridian Observation of the Sun for Apparent Noon.

Observer: Geo. F. Rigby

Recorder: LeRoy V. Wollney

Date: September 9, 1944. Instrument: W. & L. E. Gurley No. 2350

September 9, 1944, in latitude 42° 32' 24" N., and longitude 119° 46' 30" W., with the telescope in the meridian and elevated to the sun'e altitude, I obeerve the sun'e transit for time, noting the watch time of transit of each limb:
Mean watch time of apparent noon = $12^h\ 55^m\ 52^s$
Watch fast of local mean time = $58^m\ 42^s$

Field Record.

Setting:		90° 00' 00"
	ø ∠ (-)	42° 32' 24" N.
	d ∠ (+)	5° 08' 28" N.
	v ∠	52° 36' 04"

Watch time of traneit of W. limb	=	$12^h\ 54^m\ 48^s$
Watch time of traneit of E. limb	=	12 56 56
Watch time of apparent noon	=	$12^h\ 55^m\ 52^s$
Apparent noon = $12^h 00^m\ 00^s$		
Equa. of time = - 2 50		
Local mean time of apparent noon	=	11 57 10
Watch fast of local mean time	=	$58^h\ 42^m$

C - 2

MERIDIAN OBSERVATION OF THE SUN FOR APPARENT NOON.

Date.	Observer.	Instrument.
Sept. 6, 1944.	Otis O. Gould.	Buff and Buff No. 17998.

$V \neq 90° - \phi \pm D$

Final field notes.

Sept. 6, 1944, in camp located in sec. 5, T. 25 S., R. 3 W., Will. Mer., Oregon, in latitude 43° 25' 30" N., and longitude 123° 04½'W., with my watch set for pacific standard time as checked by radio time signals, with the telescope in the meridian and elevated to the sun's altitude, I observe the sun's transit for time, noting the watch time of transit of each limb:

Mean watch time of apparent noon 12h 10m 30s
Watch fast of local mean time 12m 19s

Field record.

Setting:			
			90°00'
	$\phi \neq$	(-)	43°25½'N.
	$D \neq$	(+)	6°16' N.
	$V \neq$		52°50½'

Watch time of transit, W. limb		= 12h 09m 26s
Watch time of transit, E. limb		= 12h 11m 34s
Watch time of apparent noon		= 12h 10m 30s
Apparent noon	12h 00m 00s	
Equation of time	1m 49s	
Local mean time of apparent noon		= 11h 58m 11s
Watch fast of local mean time		= 12m 19s

Meridian Observation of a Star for Time.

Star:	25/49 (Alpha) Aquilae	0.9 (Altair)	Declination 8° 43' 24" N.

Date	Observer: Norman D. Price	Instrument
October 26, 1944	Recorder: George F. Rigby	Buff No.17998

On a meridian previously determined at my station, in the SW¼ of NW¼ of sec. 25, T. 1 N., R. 1 E., of Willamette Meridian, Oregon, in latitude 45° 32' 30" N., and longitude 122° 39' W., with the telescope in the meridian and elevated to the star's altitude, to check the reading of my watch which indicates approximate Pacific war time, I observe the star's transit for time as follows:

Watch time of star's transit----------6h 37m 55s p.m.

Theoretical time for transit of star for this date is derived as follows:

6h 07m 36s p.m., Oct. 16 transit of star, Greenwich Meridian, civil date and mean time.
39m 18s subtract, correction to Oct. 26.
5h 28m 18s p.m., Oct. 26 transit of star, Greenwich Meridian, civil date and mean time.
1m 20s subtract, correction for longitude of this station.
5h 26m 58s p.m.,l.m.t. Oct. 26 transit of star.
1h 10m 36s add, correction to Pacific war time.
6h 37m 34s p.m.,P.W.T. Oct. 26 transit of star.

Watch time of star's transit-----------6h 37m 55s p.m.
Theoretical time of star's transit-----6h 37m 34s p.m.
Watch fast of Pacific war time------- 21s.

Stellar Meridian Transit Observation for Time.

Observer: Geo. F. Rigby

Recorder: Norman D. Price

Date: October 25, 1944. Instrument: W. & L. E. Gurley No. 2350

October 25, 1944, in latitude 45° 32' 30" N., and longitude 122° 39' 00" W., with the telescope in the meridian and elevated to the star's altitude, I observe the time of transit of the star (Epsilon) Pegasi to check the reading of my watch which reads approximate Pacific War Time.

Star: 26/52 2.5

(Epsilon) Pegasi

+9° 37.4'

Watch time of transit		8^h 35^m 17^s
Gr. m.t. star's transit Oct. 16	8^h 01^m 00^s	
Reduce to " 25 -	35 24	
Reduce to long. 122° 39' W. -	1 20	
Star's transit, l.m.t.	7^h 24^m 16^s	
Reduce to P.W.T.	1 10 36	
Pacific War Time of star's transit		8^h 34^m 52^s
Watch fast of Pacific War Time		0^m 25^s

Meridian Altitude Observation of the Sun for Latitude

Example of meridian altitude observation of the sun for latitude, ~~sun~~ south declination:

October 27, 1944.

Observer: George W. Johnson. Instrument: Buff No. 14187.

Collimator test of vertical circle, September 5, 1944:

Telescope direct, sights OK at zero		0° 00' 00"
Telescope reversed, sights high at zero		0° 01' 00"
Mean index correction		-0' 30"
At true vertical angle	+ 40° 00' 00"	- 40° 00' 00"
Mean corrected value	40° 00' 30"	40° 00' 00"
Mean index correction	-0' 30"	0' 00"

Final Field Notes

October 27, 1944, at the cor. of secs. 32 and 33, on the S. bdy. of T. 1 S., R. 27 W., 5th Prin. Mer., Arkansas, in approximate latitude 34° 36' N., and longitude 93° 54' W., I make a meridian observation of the sun for latitude, observing the altitude of the sun's lower limb with the telescope in direct position, reversing the telescope and observing the sun's upper limb:

Apparent time of observation, noon	= 12h 00m 00s
Mean observed altitude	= 42° 30' 00"
Reduced latitude	= 34° 36' 03"

Field Record.

Setting:

		90° 00'
Latitude	≠ (-)	34° 36' N.
Declination	≠ (-)	12° 55' S.
	v ≠	42° 29'
Lower limb		42° 13'
Upper limb		42° 45'

Observed alt., lower limb, tel. dir.	= 42° 13' 30"
Observed alt., upper limb, tel. rev.	= 42° 46' 30"
Mean observed altitude, v	= 42° 30' 00"
Transit index correction	= " -0' 30"
Refraction (El.=1000 ft.; temp.=40° F.)	= " -1' 03"
Parallax	= +0' 08"
	h = 42° 28' 35"

Decl., Gr. app. noon =	12° 50' 05.0"S.	
Reduced to long. 93° 54'		
6.26 x 50.69" = 317.3"=	- 5' 17.3"	
	12° 55' 22.3" S.	12° 55' 22" S

Latitude = 34° 36' 03" N. = 90° - decl. - h = 34° 36' 03"

90° 00' 00"

D - 2

Date - May 30, 1944 Observer - F. W. Williamson Instrument - W. and L. E. Gurley No. 38105

May 30, 1944, at U.S.L.M. No. 2383, Iliamna, Alaska, in approximate longitude 154°51' West, I make a meridian observation of the sun for latitude, observing the altitude of the sun's lower limb with telescope in direct position, reversing the transit and observing the sun's upper limb.

Apparent time of observation noon	12 h 00 m 00 s
Mean observed altitude	52°07'15"
Reduced latitude	59°44'51"

Field Record.

observed altitude lower limb direct	51°51'00"
observed altitude upper limb reversed	52°23'30"
Mean altitude suns center	52°07'15"
	40"
Altitude corrected for parallax and refraction	52°06'35"
Declination apparent noon Greenwich May 30	21°47'37"
correction for longitude 154°51' or 10.32 hrs.	3'49"
	21°51'26"

90°00'00"
21°51'26"
111°51'26"
52°06'35"
59°44'51" North latitude.

Example.

Date: September 10, 1944. Instrument: Gurley No. 371613.

Observer: Russell C. MacDonald.

Sec. 75: <u>Meridian Observation of the Sun for Time and Latitude.</u>

September 10, 1944, in approximate latitude 41° 35' N., and longitude 109° 58.3' W., and at an elevation of 6500 feet above sea level, I make a meridian observation of the sun for time and latitude, observing simultaneously the altitude of the sun's lower limb and the transit of the sun's west limb, reversing the telescope and observing the sun's upper limb and the transit of the sun's east limb.

Mean observed altitude	53° 11' 45"
Reduced latitude	41° 35' 09"
Mean watch time of observation	11h 57m 22s
Watch fast of local mean time.........	33s

Sun's declination, Greenwich apparent noon, Sept. 10, ...	4° 53' 18.7" N.
Reduced to longitude 109° 58.3' W., 7.332h x 56.88" = 416" =	6' 56.0" (S.)
The sun's apparent declination	4° 46' 22.7" N.

Setting: $v = 90° - \phi + d$

	90° 00'	Lower limb	52° 55'
ϕ = (-)	41° 35'	Upper limb	53° 27'
d = (+)	4° 46'		
v =	53° 11'		

Position of Telescope	Position of Sun	Watch Time of Transit	Observed Vertical Angle
Direct Reversed		11h 56m 18s 11h 58m 26s	52° 55' 30" 53° 28' 00"

Mean	11h 57m 22s	53° 11' 45"
Refraction, corr. to elev. of 6500 ft. 43" x 0.8(coeff.)		- 34"
Parallax		+ 05"
h		53° 11' 16"
decl. = 4° 46' 23"; 90° + d		94° 46' 23"
ϕ = 41° 35.15' N.; 90° + d - h		41° 35' 07"

Equation of time, Sept. 10, 1944, Greenwich noon ......(subt)	3m 03.9s
Correction to noon, long. 109° 58.3' W., ...	06.7s
Equation of time	3m 11s
Watch time of apparent noon	11h 57m 22s

Apparent noon	12h 00m 00s	
Equation of time ...	— 3m 11s	
Local mean time of apparent noon		11h 56m 49s
Watch fast of local mean time		33s

E - 2

MERIDIAN OBSERVATION OF THE SUN FOR TIME AND LATITUDE.

Date: Aug. 27, 1944 Observer: A. W. Brown Instrument: Buff No. 23815
Recorder: W. H. Teller
Timer: W. H. Teller

FINAL FIELD NOTES

Aug. 27, 1944, in camp in sec. 19, T. 45 N., R. 5 E., N.M.P.M., Colo. in latitude 38° 08' N., longitude 106° 21.1' W., barometer 22.1 ins. and approx. temperature 77 °F., I make a meridian observation of the sun for time and latitude, observing simultaneously the altitude of the sun's upper limb and time of transit of the sun's west limb; then reversing the telescope and observing simultaneously the altitude of the sun's lower limb and time of transit of the sun's east limb.

Mean observed altitude = 61° 47' 00"

Reduced latitude $(90° - h + \delta)$ = 38° 08' 13.5"

Mean watch time of obsn. (M.W.T), = 1h7m2s.

Watch fast of local mean time = 1h5m40s.

FIELD RECORD

Setting 90° 00'

ϕ ≠ 38° 08' N.

δ ≠ 9° 55' N.

v ≠ 61° 47' $(90° - \phi + \delta)$

Upper limb: 62° 03' (v + 16')
Lower limb: 61° 31' (v − 16')

FIELD OBSERVATION

Telescope	Sun	Watch time of sun's transit (m.w.t.)	Observed vertical angle	Barometer ins.	Temp. °F.
Direct Reversed		1h5m58s., p.m. 1h8m6s., p.m.	62° 03' 00" 61° 31' 00"	22.1	77
Mean		1h7m2s., p.m.	61° 47' 00"		

LATITUDE CALCULATION

Observed vertical angle = 61° 47' 00"
Refraction, (30.9 × .75 × .95) = 0° 0' 22"
Parallax = 4.0"

h = 61° 46' 42.0"

Sun's decl., Gr. app. t., noon = 10° 01' 08.6" N.
Diff. for 1 hr. = − 52.64"
Reduction to long. 106° 21.2' = 6' 13.1" S.

δ, decl. at time of observation = 9° 54' 55.5" N.

$\phi = 90° - h + \delta$ = 38° 8' 13.5" N.

TIME CALCULATION

Watch time of apparent noon = 1h7m2s., p.m.
Apparent noon = 12h00m00s.
Equation of time = 1m22s.
Local mean time of app. noon = 12h1m22s., p.m.
Watch fast of l.m.t.: 1h5m40s.

E - 3

MERIDIAN OBSERVATION OF A STAR
FOR TIME AND LATITUDE.

Date; Aug.29, 1944 Observer:A.W.Brown Instrument:Buff No 23815
Recorder:W.H.Teller
Timer : W.H.Teller

FINAL FIELD NOTES

Aug. 29, 1944, in camp in sec. 19, T. 45 N., R. 5 E., N.M.P.M., Colo., in latitude 38° 08' N., longitude 106° 21.1' W., barometer 22.1 ins. and approx. temperature 50° F., I make a meridian observation of the star 24/48, sigma sagittarii, observing simultaneously the altitude and time of transit, then reversing the transit telescope and observing the altitude.

Mean observed altitude = 25° 31' 30"

Reduced latitude (90 − h − δ) = 38° 07' 58"

Watch time of observation (m.w.t.) = 9h 24m 42s. p.m.

Watch fast of local mean time = 1h 6m 0s.

FIELD RECORD

Setting 90° 00'

ø ≠ 38° 08' N.

δ ≠ 26° 22' S.

v ≠ 25° 30'

t ≠ 9h 25m p.m., m.w.t.

OBSERVATION

Telescope	Watch time of star's transit (M.W.T.)	Observed vertical angle	Barometer ins.	Est. Temp. °F.
Direct	9h 24m 42s	25° 31' 30"	22.1	60
Reversed		25° 31' 30"	22.1	60
Mean	9h 24m 42s	25° 31' 30"	22.1	60

LATITUDE CALCULATION

Observed vertical angle = 25° 31' 30"
Refraction (2.0 × .75 × .98 = − 1' 28"

h = 25° 30' 02"

δ sigma sagittarii = 26° 22' 00" S.
ø = 90° − h − δ = 38° 07' 58" N.

TIME CALCULATION

Watch time of star's transit = 9h 24m 42s.
Gr. mean time of star's tran. = 8h 19m 54s.
Correction for long. 106°21.1' = − ~~1m 12s.~~ − $1^m\ 09^s$

L.M.T. of star's transit = ~~8h 18m 42s.~~ $8^h\ 18^m\ 45^s$

Watch fast of L.M.T.: ~~1h 6m 00s.~~ $1^h\ 5^m\ 57^s$

Example.

Date: September 10, 1944. **Instrument:** Gurley No. 371613.

Observer: Russell C. MacDonald.

Sec. 75A: Meridian Transit of a Star for Watch Correction in Local Mean Time, and Latitude.

September 10, 1944, in approximate latitude 41° 35' N., and longitude 109° 58.3' W., and at an elevation of 6500 feet above sea level, I make a meridian observation of σ Sagittarii for watch correction in local mean time, and latitude, observing simultaneously the altitude and transit of the star, reversing the telescope and observing the altitude of the star.

Mean observed altitude	22° 04' 45"
Reduced latitude	41° 35' 09"
Watch time of star's transit	7h 31m 56s p.m.
Watch fast of local mean time ...	32s

σ Sagittarii: declination 26° 22.0' S., Magnitude 2.1 No. 24/48

Greenwich mean time of transit	8h 08.0m p.m.
Correction to September 10	- 35.4m
Sidereal conversion to longitude 109° 58.3' W.	- 1.2m
Local mean time of star's transit	7h 31.4m p.m.

Setting: $v = 90° - \phi - d$

$$\begin{array}{lr} & 90°\ 00' \\ \phi = (-) & 41°\ 35' \\ d = (-) & \underline{26°\ 22'} \\ v = & 22°\ 03' \end{array}$$

Position of Telescope	Watch Time of Transit	Observed Vertical Angle
Direct Reversed	7h 31m 56s p.m.	22° 04' 30" 22° 05' 00"

Mean	7h 31m 56s p.m.	22° 04' 45"
Refraction, corr. to elev. of 6500 feet, 2' 22" x 0.80 (coeff.)		- 1' 54"
h		22° 02' 51"
d = 26° 22' S.; 90° - d		63° 38' 00"
φ = 41° 35' N.; 90° - d - h		41° 35' 09"

Watch time of star's transit	7h 31m 56s p.m.
Local mean time of star's transit	7h 31m 24s p.m.
Watch fast of local mean time	32s

F - 1

Example of altitude observation of Polaris at Upper Culmination for latitude:

Observer: Glenn R. Haste

Final field notes.

August 31, 1944, in approximate latitude 33° 23' N., and longitude 107° 11' 38" W., I make an altitude observation on Polaris at upper culmination for latitude, making four observations, two each with the telescope in direct and reversed positions.

Watch set correct mountain war time by radio signal.	
Mean watch time of observation	= 4h 17m 01s a.m.
Mean observed vertical angle	= 34° 24' 52"
Reduced latitude	= 33° 23'.4 N.

Field record.

$$
\begin{array}{rl}
\text{Setting:} & 90^\circ\ 00' \\
d \neq & 89^\circ\ 00' \\
\hline
90^\circ - d \neq & 1^\circ\ 00' \\
\phi \neq & 33^\circ\ 23' \\
\hline
v \neq & 34^\circ\ 23' \quad \phi + (90^\circ - d)
\end{array}
$$

Gr. U. C. of Polaris, August 31, 1944	= 3h 09.5m
Reduced to longitude 107° 11'.6 W.	= - 1.2
	3h 08.3m
L. M. T. of U. C. of Polaris, August 31	= 3h 08m 24s
War time fast of standard time	= 1h 00m 00s
Correction for longitude 107° 11' 38"	= 0h 08m 47s
Computed watch time of upper culmination	= 4h 17m 11s

Telescope	Watch time.	Vertical angle.
Direct ----------------------------	4h 13m 03s	34° 24' 00"
Reversed --------------------------	4h 15m 01s	34° 26' 00"
Reversed --------------------------	4h 19m 00s	34° 26' 30"
Direct ----------------------------	4h 21m 00s	34° 23' 00"
Mean ----------------------	4h 17m 01s	34° 24' 52"
Refraction (cor. for elev.3600')-		- 1' 15"
	h =	34° 23' 37"
d = 88° 59' 47"; 90° - d ------------------	=	1° 00' 13"
ϕ = 33° 23'.4 = h+(90°-d) ------------------	=	33° 23' 24"

F - 2

ALTITUDE OBSERVATION OF POLARIS AT UPPER CULMINATION FOR LATITUDE

Date: Oct. 18, 1944. Observer: Elliot Bird Instrument: Buff No. 18003.

In approximate latitude 40°44' N., longitude 111°52' W., I make an altitude observation on Polaris at upper culmination for latitude, making four observations, two each with the telescope in direct and reversed positions.

Watch carries correct 105th meridian standard war time, having been compared with Western Union time signals broadcast hourly over radio station K S L, Salt Lake City, Utah.

Setting:

$$
\begin{array}{rl}
 & 90°\ 00' \\
\phi \neq & \underline{40°\ 44'} \\
90° + \phi \neq & \underline{130°\ 44'} \\
\delta \neq & \underline{89°\ 00'} \\
v \neq & 41°\ 44' = \phi + (90° - \delta)
\end{array}
$$

Greenwich U.C. of Polaris Oct. 18, 1944	= 0h 1.2m a.m.
Reduction for longitude	= -1.2m
Local mean time U.C. of Polaris Oct. 17	= 0h 0.0m (midnight)
Correction for longitude 111°52' W.	= +27m 28s
Correction for war time	= 1h 00m 00s
Computed watch time of U.C. Oct. 18	= 1h 27m 28s a.m.

Telescope	Watch time	Vertical angle
Direct..................	1h 23m 4s	41° 44' 30"
Reversed................	1h 27m 7s	41° 45' 00"
Reversed................	1h 30m 2s	41° 45' 00"
Direct..................	1h 33m 7s	41° 44' 30"
Mean................	1h 28m 20s	41° 44' 45"
Refraction....................................		-1 5
	h	=41° 43' 40"
δ = 89°00'03"; 90° - δ		= -59' 57"
ϕ = 40°43'.7 N. = h + δ - 90°		=40° 43' 43"

MEMO;

Refraction corrected for elevation above sea level (4500 ft.) as follows:

Observed vertical angle	= 41° 44' 45"
Refraction (1' 05" X .86 = 56")	= -56
h	= 41° 43' 49"
δ = 89° 00' 03"; 90° - δ	= 59 57
ϕ = 40° 43.9' N. = h+δ-90°	= 40° 43' 52"

Example of Altitude Observation of Polaris at any Hour Angle for Latitude, Sec. 133-A, making use of the table given in the Ephemeris:

Final Field Notes.

June 23, 1944, in the SE. ¼ of sec. 16, T. 7 N., R. 3 E., Salt Lake Meridian, in approximate latitude 41° 20' N., and longitude 111° 37' W., I make an altitude observation on Polaris at an hour angle for latitude, making four observations, two each with the telescope in direct and reversed positions:

Mean observed vertical angle	42° 00' 30"
Mean watch time of observation, a. m.	4h 46m 38s
Watch reads correct 105th meridian standard time, after deducting 1 hour from war time, as determined by radio signals	00m 00s
Reduced latitude	41° 20' 37"

Field Record

Hour angle observation of Polaris for latitude:

Telescope	Vertical angle.	Watch time.
Direct............................	41° 58' 30"	4h 44m 45s a. m.
Reversed..........................	41 59 30	4 45 50
Reversed..........................	42 01 30	4 47 20
Direct............................	42 02 30	4 48 37
Mean..............................	42° 00' 30"	4h 46m 38s a. m.
Watch fast of local mean time or correction for longitude 111° 37' W.		26 28
L. M. T. of observation, June 23, 1944		= 4h 20m 10s a. m.
		= 4h 20.2m a. m.
Gr. U. C. of Polaris, same date	= 7h 39.4m a. m.	
Reduced to longitude 111° 37' W.	= -1.2m	= 7h 38.2m a. m.
Less L. M. T. of observation same date		4 20.2
Argument for table for hour angle of Polaris east of mer.		= 3h 18.0m
Declination of Polaris		= +88° 59' 39"

Mean time hour angle	Primary adjustment, subtractive, Polaris above the pole. Declination. 88° 59' 40"
3h 17.5m	0° 38' 53"
3 18.0	0 38 47
3 23.4	0 37 39

Mean observed vert. ang.	v = 42° 00' 30"	
Correction for refraction	= -01 04	See memo. below.
	h = 41° 59' 26"	
Primary adjustment to elevation of pole	= -0° 38' 47"	
Supplemental correction	= -0° 0' 02"	
Latitude of sta.	= 41° 20' 37"	

Determination of the Azimuth from the Mean Time Hour Angle obtained in the Altitude observations of Polaris for Latitude, given in above example:

Mean time hour angle	Azimuth of Polaris. Mean Declination. + 89° 00' 00". Latitude 40°	41° 20' 37"	42°	Correction additive for declination 88° 59' 40"
3h 9.5m	58.3'	59.6'	60.2'	0.3'
3 18.0		61.6		0.3
3 19.5	60.6	61.9	62.5	0.3

Azimuth of Polaris = 1° 1.6' + 0.3' = 1° 1.9' = 1° 1' 54"

J. C. Clark

Analytical reduction for Latitude, using the Mean Time Hour Angle and Altitude given in the example on sheet No. 2:

In order to proceed with the analytical reduction, it is convenient to begin with an angle a, computed from the equation:

$\text{Tan } a = \dfrac{\tan \delta}{\cos t}$, in which equation the factor "cos t" becomes negative for hour angles exceeding 90°, whereupon a will exceed 90°.

t = sideral hour angle counting from the meridian.

$\delta = 88° \; 59' \; 39"$. $\qquad h = 41° \; 59' \; 26"$.

The latitude is derived from the equation:

$$\text{Cos} \, (\phi - a) = \frac{\sin a \sin h}{\sin \delta}$$

The above example of the field observation is reduced as follows:

Mean time hour angle	3h 18.0m	log tan δ = 11.7555520	log sin a = 9.9999719
Reduced to sideral hour angle add	33s	" cos t = 9.8113212	" " h = 9.8254313
		" tan a = 1.9442308	19.8254032
Sideral hour angle t	3h 18m 33s		" " δ = 9.9999331
			log cos ($\phi - a$) = 9.8254701
Same reduced to angular measurement t = 49° 38' 15"		a = 89° 20' 55"	

By inspection
$\phi - a$ is a
negative angle.

$\phi - a$ = −48° 0' 17"
a = 89° 20' 55"

Latitude of station...... ϕ = 41° 20' 38"

MEMO:

The refraction correction is adjusted for elevation of station (+ 6800 ft.) as follows:

Mean observed vertical angle, v	=	42° 00' 30"
Refraction corr. (1' 04" X .79 = 51")	=	-51
h	=	41° 59' 39"
Primary adjustment to elev. of pole	=	-38 47
Supplemental correction	=	-02
Latitude of station, ϕ	=	41° 20' 50" N.

Latitude by formulas as above, using corrected value for "h":

M.t.h.a.	$3^h \; 18^m \; 00^s$	log tan δ = 11.7555520	log sin a = 9.9999719
Red. to sid. h.a.	+33	" cos t = 9.8113212	" " h = 9.8254618
Sidereal h.a.	$3^h \; 18^m \; 33^s$	" tan a = 1.9442308	9.8254337
t = 49° 38' 15"		a = 89° 20' 55"	" " δ = 9.9999331

log cos (ϕ-a) = 9.8255006
(ϕ-a) = -48° 00' 04"
a = 89° 20' 55"

ϕ = 41° 20' 51" N.

J. C. Clark

Example of hour angle observation of Polaris for latitude:

Observer: Glenn R. Haste.

Final field notes.

Aug. 31, 1944, at the standard cor. of Tps. 10 S., Rs. 3 and 4 W., in longitude 107° 11' 38" W., I make an hour angle observation of Polaris for latitude, reading two vertical angles, one each with the telescope in direct and reversed positions:

Mean observed vertical angle	34° 15' 45"
Mean watch time of observation, a.m.	6h 22m 32s
Watch fast of l.m.t., determined by radio signal reading mountain war time	1h 08m 47s
Reduced latitude	33° 23' 15"

Field record.

Hour angle observation of Polaris for latitude:

Telescope.	Vertical angle.	Watch time.
Direct ----------------	34° 16'	6h 22m 10s a.m.
Reversed --------------	34° 15' 30"	6h 22m 54s
Mean -------------	34° 15' 45"	6h 22m 32s a.m.
Watch fast of local mean time ------------		1h 08m 47s

L. M. T. of observation, Aug. 31, 1944		= 5h 13m 45s a.m.
		= 5h 13.7m a.m.
Gr. U. C. of Polaris, same date	3h 09.5m a.m.	
Reduced to longitude 107° 11' 38" W.	- 1.2	= 3h 08.3m a.m.
Hour angle of Polaris west of the meridian		= 2h 05.4m
Declination of Polaris		+ 88° 59' 46.9"

Mean time hour angle.	Primary adjustment, subtractive, Polaris above the pole. Declination.		
	+88° 59' 40"	+88° 59' 46.9"	+89° 00' 00"
1h 59.7m	0° 52' 07"	0° 52' 01"	0° 51' 50"
2h 05.4		0° 51' 13"	
2h 11.6	0° 50' 27"	0° 50' 21"	0° 50' 10"

Mean observed vertical angle, v	= 34° 15' 45"
Correction for refraction, (elev. 3600 ft. above sea level.)	= 1' 15"
h	= 34° 14' 30"
Primary adjustment to elevation of pole.	- 0° 51' 13"
Supplemental correction	- 0° 00' 02"
Latitude of station.	= 33° 23' 15"

In order to proceed with the analytical reduction, it is convenient to begin with an angle a, computed from the equation: $\text{Tan } a = \frac{\tan d}{\cos t}$, in which equation the factor "cos t" becomes negative for hour angles exceeding 90°, whereupon a will exceed 90°.
The latitude may then be derived from the equation:

$$\text{Cos}(\phi - a) = \frac{\sin a \sin h}{\sin \delta}$$

The above example of a field observation is reduced as follows:

Mean time h.a.	2h 05.4m	log tan d = 1.7565011	log sin a = 9.9999514
		" cos t = 9.9311136	" " h = 9.7502651
Red. to sideral h. a.	+ 0.3	" tan a = 1.8253875	9.7502165
Sideral h.a.	2h 05.7m		" " d = 9.9999337
		a = 89° 08' 37"	
Same, red. to ang. meas.,	t = 31° 25' 30"		log cos (ϕ-a) = 9.7502828

By inspection it will be seen that ϕ-a is a negative angle.	ϕ-a = -55° 45' 24"
	a = +89° 08' 37"
Latitude of station ------	ϕ = 33° 23' 13"

H - 1

Date: Nov. 2, 1944. Observer: R. Y. Lyman. Instrument : Gurley No. 371657.

Field record.		Final field notes.
Nov. 2, 1944, Gr. E. E. of Polaris, lat. 40°	5^h 02.6^m p.m.	Nov. 2, 1944, at the cor. of secs. 1, 2, 11 and 12, T. 7 N., R. 1 E., Prin. Mer., Montana, latitude 46° 24' N., longitude 111° 33' W., at 5^h 02.2^m p.m., l.m.t., I observe Polaris at eastern elongation, making four observations, two each with the telescope in direct and reversed positions, and mark the mean point in the line thus determined, on a peg driven in the ground, 5 chs. N.
Reduced to long, 111° 33' W.	- 1.2^m	
Reduced to lat. 46° 24' N.	+ 0.6	
L.m.t. of E. E. of Polaris	5^h 02.2^m p.m.	
Watch fast of l.m.t. by radio time signals	1^h 26.2^m	
Watch time of E. E.	6^h 28.4^m p.m.	
Telescope.	**Watch time.**	Azimuth of Polaris at eastern elongation 1° 26' 47".
Direct....................	6^h 28^m 20^s p.m.	At 6^h 35^m p.m., M.W.T., I lay off the azimuth of Polaris 1° 26' 47" and make a meridian mark on a second peg, 8.33 feet to the west of the mean point in the line determined by observation; I verify the angle by a vernier reading of the instrument.
Reversed..................	6 28 30	
Reversed..................	6 29 30	
Direct....................	6 30 20	
Mean..................	6h 29^m 40^s p.m.	

Declination of Polaris 89° 00' 09.3" N.

	Declination.		
Latitude	89° 00' 00"	89° 00' 09"	89° 00' 10"
		Azimuth	
46° 00' 00"	1° 26' 23"	1° 26' 09"	1° 26' 09"
46 24 00		1 26 47	
47 00 00	1 27 59	1 27 43	1 27 44

H - 2

Elongation Observation of Polaris, Observing Program "a."

Observer: Geo. F. Rigby

Recorder: LeRoy V. Wollney

Date: September 9, 1944. Instrument: W. & L. E. Gurley No. 2350

Field Record

Sept. 9, 1944, Gr. E.E. of Polaris, lat. 40°	$8^h\ 34^m\ 36^s$
Red. to long. 119° 46' 30" W.	− 1 19
Red. to lat. 42° 32' 24" N.	+ 0 21
L.M.T. of E.E. of Polaris	$8^h\ 33^m\ 38^s$
Watch fast of l.m.t.	0 58 42
Watch time of E.E.	$9^h\ 32^m\ 20^s$

Telescope	Watch time
Direct	$9^h\ 29^m\ 50^s$ p.m.
Reversed	9 32 07
Reversed	9 33 34
Direct	9 35 36
Mean	9 32 47 p.m.

Declination of Polaris 88° 59' 49"

Latitude	Declination 88°59'40"	Declination 88°59'49"	Declination 88°59'50"
	Azimuth		
42°00'00"	1°21'11"	1°20'59"	1°20'58"
42°32'24"		1°21'41"	
43°00'00"	1°22'30"	1°22'17"	1°22'16"

Final Field Notes

Sept. 9, 1944, in camp in SE¼NW¼ sec. 15, T. 35 S., R. 25 E., Willamette Meridian, Oregon, in latitude 42° 32' 24" N., and longitude 119° 46' 30" W., at $8^h\ 33^m\ 38^s$ p.m., l.m.t., I observe Polaris at eastern elongation, making four observations, two each with the telescope in direct and reversed positions, and mark the mean point in the line thus determined, on a peg driven firmly in the ground, 10 chs. N.

Azimuth of Polaris at eastern elongation 1° 21' 41"

Sept. 10: I lay off the azimuth of Polaris, 1° 21' 41", and make a meridian mark on a second peg, 23.77 lks. (15.69 ft.) to the west of the mean point in the line determined by the observation; I verify the angle by a vernier reading of the instrument.

H - 3

July 9, 1944.

Observer: Walter H. Good
Recorder: Ditto
Time : Ditto

Instrument:
Gurley No. 38103.

Field Record.

July 9, 1944: Gr. E. E. of Polaris, lat. 40°	0h 41.2m, a.m.
Red. to long. 113° 35' W.	- 01.25
Red. to lat. 42° 25' N.	+ 0.35
L.M.T. of E.E. of Polaris	0h 40.3m a.m.
Watch fast of L.M.T.	34.7
Watch time of E.E.	1h 15.0m a.m.

Telescope.	Watch Time.
Direct.	1h 12m 08s
Reversed	1h 14m 12s
Reversed	1h 15m 36s
Direct	1h 17m 00s
Mean	1h 14m.44s a.m.

Declination of Polaris 88° 59' 38.4s

Logcos 88° 59' 38.4" = 8.244453
" 42° 25' = 9.868209

Logsin A = 8.376244
A= 1° 21' 46"

MEMO:
Reduction formulae used above-

$$\sin A = \frac{\cos \delta}{\cos \phi}$$

Final field notes.

July 9, 1944, at the corner of Tps. 11 and 12 S., Rs. 24 and 25 E., Boise Meridian, in lat. 42° 25' N., long. 113° 35' W., at 0h 40.3m., a.m., l.m.t., I observe Polaris at eastern elongation, making four observations, two each with the telescope in direct and reversed positions, and mark the mean point in the line thus determined, on a peg driven firmly in the ground, 8.00 chs. N.

Azimuth of Polaris at eastern elongation= 1° 21' 46".

July 10: I lay off the azimuth of Polaris, 1° 21' 46", to the west, and mark the meridian thus determined by a tack in a peg driven firmly in the ground, 8.00 chs. N.

H - 4

Sept. 4, 1944.

William H. Teller, observer;
Donald Skinner, recording.

Buff No.18001
General Land Office
solar transit.

Polaris at elongation, observing program "b".

Field Record

Sept. 4, 1944, Gr. E. E. of Polaris, lat. 40°	= 8h 54'.2 p.m.
Red. to long. 106° 22' W.	= - 1'.3
Red. to lat. 37° 45' N.	= 0'.2
L. M. T. of E. E. of Polaris	8h 53.1 p.m.
Watch fast of L. M. T.	5.3
Watch time of E. E. pf Polaris	8h 58'.4 p.m.

Telescope	Watch Time	Deflection angle
Direct	8h 56m 38s	1° 16' 00"
Reversed	8h 57m 28s	1° 16' 00"
Direct	8h 57m 43s	1° 16' 00"
Reversed	8h 58m 56s	1° 16' 00"-
Mean	8h 57m 41s	1° 16' 00"

Declination of Polaris = 88° 59' 48" N.

Latitude	Declination 88° 59' 40"	Declination 88° 59' 48"	Declination 88° 59' 50"
37° 00'	1° 15' 33"	1° 15' 23"	1° 15' 20"
37° 45'		1° 16' 08"	
38° 00'	1° 16' 34"	1° 16' 24"	1° 16' 21"

Final Field Notes

Sept. 4, 1944, at a point near the SE. cor. of T. 41 N., R. 5 E., N. M. P. M., in Colorado, in latitude 37° 45' N., longitude 106° 22' W., at 8h 57'.7m, M. W. T., I observe Polaris at E. Elongation, making four observations, two each with the telescope in direct and reversed positions, reading the deflection angle to a point on cliffs eight chains N. of my station, east to Polaris:

Azimuth of Polaris at eastern elong.	= 1° 16' 08"
Mean deflection angle	= 1° 16' 00"
True bearing of mark	= N.0°00' 08" E.

The sun at meridian passage, for time.

Sept. 4.

Watch time, sun's west limb		$12^h\ 03^m\ 10^s$
" " " east "		12 05 15
Watch time of apparent noon		$12^h\ 04^m\ 12^s$
Apparent noon	$12^h\ 00^m\ 00^s$	
Equation of time	- 1 07	L.M.T.: 11 58 53
Watch fast of local mean time		$5^m\ 19^s$

Observer Thomas W. Crawford
Transit Buff No. 10899.

H - 5

September 10, 1944, at a transit point, near the north east corner of T. 38 N., R. 7 E., New Mexico Principal Meridian, Colorado, in latitude 37° 34.'5 N., and longitude 106° 8.'4 W., at 8h 27.3m p.m., l. m. t., I observe Polaris at eastern elongation, making four observations, two each with the telescope in direct and reversed positions, reading the deflection angle from a light bulb, in a street light, about 20 chs. N., east to Polaris:

Azimuth of Polaris at eastern elongation	1° 15' 55"
Mean deflection angle	2° 02' 22"
True bearing to street light	N.0° 46' 27"W.

Sept. 10, 1944, Gr. E. E. of Polaris lat. 40°		8h 30m 42s p.m.
Red. to long. 106° 8.'4 W.	-	1 10s
Red. to lat. 37°34.'5 N.	-	15s
L. M. T. of E. E. Polaris		8h 29m 17s p.m.
Watch fast of L. M. T.		4m 32s
Watch time of E. E.		8h 33m 49s p.m.

Telescope	Watch time	Deflection angle
Direct	8h 27m 30s	2° 01' 30"
Reversed	8 30 00	2° 03' 00"
Reversed	8 32 42	2° 02' 00"
Direct	8 36 10	2° 03' 00"
Mean	8h 31m 37s	2° 02' 22"

Declination of Polaris 88° 59' 49."7

	88° 59' 40"	88° 59' 49."7	88° 59' 50"
		Azimuth	
Latitude			
37° 00'	1° 15' 33"	1° 15' 20."4	1° 15' 20"
37° 34.5		1° 15' 55"	
38° 00'	1° 16' 34"	1° 15' 21."4	1° 16' 21"

Azimuth of Polaris at elongation computed by the equation

$$\text{Sin A} = \frac{\cos d}{\cos \phi}$$

log cos d	8.2430599	
" cos ϕ	9.8990298	
Sin A	9.3440301	1° 15' 55"

H - 6

Example of observation of Polaris at elongation for azimuth, observing program "$_o$":

Field record.

May 31, 1944, Gr.m.t., E.E. of Polaris, lat. 40°	3^h 13.8^m a.m.
Red. to long. 104°17'05.5"W.	- 1.1
Red. to lat. 33°23'40.8"N.	- 0.7
L.m.t. of E.E. of Polaris	3^h 12.0^m a.m.

Declination of Polaris = 88° 59' 41.5"

Latitude	Declination		
	88°59'40"	88°59'41.5"	88°59'50"
	Azimuth		
33°00'00"	1°11'56"		1°11'44"
33 23 40.8	1 12 16	1°12'14"	1 12 04
34 00 00	1 12 47		1 12 34

Transcribed field notes.

May 31, 1944, at the closing Tp. cor., on the 2nd Standard Parallel South, in latitude 33° 23' 40.8" N., and longitude 104° 17' 05.5" W., as computed from tie to U. S. Coast and Geodetic Survey station "Comanche" in sec. 36, T. 10 S., R. 26 E., in order to verify the alignment of the east bdy. of T. 11 S., R. 26 E., I bisect Polaris, follow the motion of the star to eastern elongation, at 3^h 12^m a.m., l.m.t., and mark the direction upon a peg driven firmly in the ground 8 chs. N.; I then reverse the instrument, again bisect Polaris, and mark the direction upon the peg. Without changing the instrument in horizontal motion, I sight to Polaris to make certain that the settings were made at elongation; there appeared to be no deviation in azimuth for some 15 or 20 minutes.

I lay off the azimuth of Polaris, 1° 12' 14", to the west of the mean direction determined by the observation and set a point for the test meridian; then by direct and reversed sights, I ascertain that the angle subtended by the flag on the cor. of secs. 1, 6, 7, and 12, is 0° 13' 08", or 0° 00' 08" from the reported bearing.

Summary [illegible] program [illegible]

[illegible]

Hour Angle Observation of Polaris in Daylight with Verification of Watch Time by Noon Observation of Sun.

Date	Observer	Instrument
September 4,1944	Norman D. Price	Buff No.17998

At camp in sec. 5, T. 25 S., R. 3 W., Willamette Mer., Oregon, in lat. 43° 25' 30" N., and long. 123° 04' 25" W., to check the reading of my watch which indicates approximate standard time for the 120th meridian, I make a meridian observation of the sun for apparent noon as follows:

Watch time of sun's transit-------12h 10m 36s p.m.
Watch fast of l.m.t.-------------- 11m 44s
Same by calculated diff. in long.- 12m 18s
Watch slow----------------------- 34s

At the same station, with my watch re-set to indicate correct 120th mer. time, I make an hour angle observation of Polaris east of the meridian, making four observations, two each with the telescope in direct and reversed positions and mark the mean point in the line thus determined, on a peg driven firmly in the ground, 10 chs. northerly from my station, as follows:

Telescope	Watch Time
Direct	6h 28m 15s p.m.
Reversed	6 31 15
Reversed	6 32 25
Direct	6 34 50
Mean-----------	6h 31m 41s p.m.
Watch fast of l.m.t.----	12m 18s
L.M.T. of obsn.---------	6h 19m 23s p.m.

Nearest u.c. of Polaris, civil date and mean time, is

2h 49m 54s a.m., September 5, 1944
Sub. 1m 20s Reduction to longitude of this station
2h 48m 34s a.m., l.m.t., u.c. of Polaris
12h 00m 00s-Add
14h 48m 34s
6h 19m 23s-Subtract l.m.t. of observation
8h 29m 11s Mean time hour angle of Polaris
1m 24s-Add for reduction to sidereal time
8h 30m 35s Sidereal time hour angle of Polaris
127°38'45" Sidereal time hour angle of Polaris

$$\tan A = \frac{\sin T}{\cos\phi \tan D - \sin\phi \cos T}$$ Formula for reduction of Azimuth "A"

T = Sidereal hour angle 127° 38' 45"
ϕ = Latitude of station 43° 25' 30"
D = Decl. of Polaris 88° 59' 48"

Log cosϕ 9.861101
Log tanD 11.756633
Add------ 1.617734
Anti-log-41.470
Anti-log- 0.41985-Add
Anti-log-41.88985-of denominator
Log- 1.622110 of denominator

Log sinϕ 9.837213
Log cosT 9.785884
Add----- 9.623097
Anti-log--0.41985
Algebraic sign is minus since cosT is in 2d quadrant.

Numerator--------------------------Log sinT 9.898617
Subtract log. of denominator---------------- 1.622110
Log. tanA----------------------------------- 8.276507
Azimuth of Polaris "A"-------------------1° 04' 58"

I lay off the azimuth of Polaris 1° 05', to the west, and mark the meridian thus determined, by a tack in a peg driven firmly in the ground, 10 chs. N. of my station.

Hour Angle Observation of Polaris, Observing Program "a."

Observer: Geo. F. Rigby   Recorder: LeRoy V. Wollney

Date: September 9, 1944.   Instrument: W. & L. E. Gurley No. 2350

Field Record

Meridian observation of the sun for apparent noon:

ϕ =	42° 32' 24" N.		90° 00' 00"
d =	5 08 28 N.		37 23 56
ϕ - d =	37° 23' 56"	v =	52° 36' 04"

	Watch time
Sun's W. limb	$12^h\ 54^m\ 48^s$
" E. "	12 56 56
Watch time of app. noon	$12^h\ 55^m\ 52^s$
App. noon $12^h\ 00^m\ 00^s$	
Equa. of time - 2 50	
L.M.T. of apparent noon	11 57 10
Watch fast of l.m.t.	$58^h\ 42^m$

Hour angle observation of Polaris:

Telescope	Watch time
Direct	$7^h\ 59^m\ 32^s$ p.m.
Reversed	8 00 50
Reversed	8 02 00
Direct	8 03 20
Mean	$8^h\ 01^m\ 26^s$ p.m.
Watch fast of l.m.t.	- 58 42
L.M.T. of obsn.	$7^h\ 02^m\ 44^s$ p.m.
	$7^h\ 02.7^m$ p.m.

Gr. U.C. of Polaris, Sept. 10, 1944	$2^h\ 30.3^m$ a.m.
Red. to long. 119° 46' 30" W.	- 1.3
L.M.T., U.C. of Polaris, Sept. 10	$2^h\ 29.0^m$ a.m.
	+12
L.M.T. of obsn. Sept. 9	7 02.7 p.m.
Hour angle of Polaris east of meridian	$7^h\ 26.3^m$
Declination of Polaris	88° 59' 50"

Mean time hour angle	Azimuth of Polaris, Mean declination +89° 00' 00", Latitude 42°	42°32.4'	44°	Correction addative for declination 88° 59' 50"
$7^h18.8^m$	75.5'	76.1'	77.9'	0.2'
7 26.3		75.1		0.2'
7 28.8	74.1	74.8	76.6	0.2'

Azimuth of Polaris = 1° 15.1' + 0.2' = 1° 15' 18"

Final Field Notes

Sept. 9, 1944, in camp in the SE¼NW¼ of sec. 15, T. 35 S., R. 25 E., Willamette Meridian, Oregon, in latitude 42° 32' 24" N., and longitude 119° 46' 30" W., I make a meridian observation of the sun for apparent noon.

Watch time of obsn. = $12^h\ 55^m\ 52^s$

Watch fast of l.m.t. = $58^m\ 42^s$

At the same station, at $7^h\ 02.7^m$ p.m., l.m.t., I make an hour angle observation on Polaris east of the meridian, making four observations, two each with the telescope in direct and reversed positions, and mark the mean point in the line thus determined, on a peg driven firmly in the ground, 10 chs. N.

Watch time of obsn., mean of four readings = $8^h\ 03^m\ 26^s$ p.m.

Sept. 10, I lay off the azimuth of Polaris, 1° 15' 18", to the west, and mark the meridian thus determined, by a tack in a peg driven firmly in the ground, 8 chs. N.

1 - 4

Date: May 17, 1943. Observer: Hal D. Craig Instrument,
Recorder: M. S. Craig Buff No. 9985.

HOUR ANGLE OBSERVATION OF POLARIS, OBSERVING PROGRAM "b"

Field record.

Hour angle observation of Polaris

Telescope	Horizontal angle; Polaris to Flag	Watch time, P.M.
Direct	0° 28' 00"	7h 38m 00s
Reversed	0° 26' 00"	7h 40m 00s
Reversed	0° 25' 00"	7h 41m 00s
Direct	0° 27' 00"	7h 42m 00s
Mean ------	0° 26' 30" ------	7h 40m 15s

Watch corrected to 90th meridian standard time.

Correction for longitude ------ 4m 40s

L.M.T. of observation, May 17, 1943 ---------------- 7h 44m 55s

Gr. U.C. of Polaris, same date, 10h 6m 54s, a.m.

Reduced to longitude 88° 50' W., ---- 0m 58s 10h 05m 56s

Hour angle of Polaris, west of meridian ------------- 9h 38m 59s

Declination of Polaris 88° 59' 26.8"

Azimuth of Polaris — Mean declination 88° 59' 45" — Latitude				Correction additive for Declination 88° 59' 26.8"
Mean time hour angle	44°	45°59'	46°	
9h 38.4m	47.4'	49.0'	49.0'	00.2'
9h 39.0m		48.8'		00.2'
9h 48.4m	44.4'	45.9'	45.9'	00.2'

Azimuth of Polaris, 0° 48' 48"+ 00.2'
0° 49' 00"

Final field notes.

May 17, 1943:

At my Station in the Village of Alvin, Wis., approximately 20 chs. north of the south boundary of section 35, T. 41 N., R. 13 E., 4th Prin. Mer., Wis., in longitude 88° 50' W., and latitude 45° 59' N., having corrected my watch to the true 90th meridian standard time; at 7h 45m 25s l.m.t., I make an hour angle observation on Polaris west of the meridian, two each with the telescope in direct and reversed positions, reading the horizontal angle from a narrow strip of flag cloth placed vertically on a tree about 7.00 chs. dist., which an earlier solar observation shows to have a bearing of N. 0° 23' W.; to Polaris, at the left of the reference flag.

Watch time of observation, 7h 40m 15s, p.m.

Mean horizontal angle from Polaris to flag, 0° 26' 30"

Azimuth of Polaris west of meridian, 0° 49' 00"

True bearing of flag, N. 0° 22' 30"W.

I - 5

Example of Hour Angle Observation of Polaris for Azimuth

Observer: Carl S. Swanholm   Instrument: Buff and Buff No. 17994

September 23, 1944

Station: A point on the Navy power line near the Hawthorne terminus in section 27, T. 8 N., R. 30 E., Mount Diablo Meridian, Nevada.

Latitude: 38°31' N.   Longitude: 118°38' W., (115°= 7h40m
3°= 12m
38'= 2m32s
7h54m32s)

Long. correction, additive to Pacific standard time   5m28s

Hour angle observation on Polaris, east of the meridian: Reference to center of water tower about 3 miles distant, and west of Polaris.

Telescope	Horizontal angle from tower to Polaris	Watch time
Direct	26°25'00"	7h24m30s p.m.
Reversed	26°25'30"	7 28 30
Reversed	26°26'00"	7 33 30
Direct	26°26'30"	7 38 00
Mean	26°25'45"	7 31 15 p.m.

Watch fast of Pacific std. time	- 1h03m08s
Longitude correction	+ 5m28s
L. M. T. of observation	6h33m43s p.m.
	6h33.7m p.m.

Greenwich U. C. of Polaris, September 24th- - - - - -	1h35.4m a.m.
Correction for longitude	- 1.3m
L. M. T. of U. C. - - - - - - - - - - - - - - - - - -	1h34.1m a.m.
	+12h
	13h34.1m
L. M. T. of observation- - - - - - - - - - - - - - - -	6h33.7m
Hour angle of Polaris, east of the meridian	7h00.4m

Declination of Polaris, September 23, 1944- - 88°59'54.0"

Azimuth of Polaris

Mean time hour angle	Mean declination 89°00'00" Latitude 38°00'	38°31'	40°00'	Correction to azimuth Additive for declination 88°59'54"
6h58.9m	73.3'	73.8'	75.4'	
7h00.4m		73.6'		0.1'
7h08.8m	72.3'	72.8'	74.4'	

Azimuth of Polaris, declination 89°00'00"	73.6'
Correction to declination 88°59'54.0"	+ 0.1'
	73.7
Azimuth of Polaris- - - - - - - - - - - - - - - - - -	N. 1°13'42" E.
Horizontal angle to west of star	26°25'45"
True bearing of reference point- - - - - - - - - - - -	N. 25°12'03" W.

- -

Meridian observation of the sun for apparent noon:

September 24, 1944

	Watch time
Tangent to Sun's W. limb	12h48m35s p.m.
Tangent to Sun's E. limb	12 50 46 p.m.
Watch time of apparent noon	12 49 41 p.m.

Apparent noon	11h59m60s	
Equation of time	- 7 59	
L.M.T. of apparent noon	11h52m01s a.m.	11 52 01 a.m.
Watch fast L.M.T.		0h57m40s
Long. correction		+ 5 28
Watch fast 120 Mer. S. time		1h03m08s

- -

Hour angle observation on Polaris, west of the meridian; Reference to same point selected for observation of preceding evening.

Telescope	Horizontal angle from tower to Polaris		Watch time	
1st set:				
Direct	23°59'15"		7h09m20s a.m.	
Reversed	23°59'30"		7 13 00s a.m.	
Mean		23°59'22"		7h11m10s a.m.
2nd set:				
Direct	23°58'45"		7h17m00s a.m.	
Reversed	23°58'30"		7 19 00 a.m.	
Mean		23°58'38"		7h18m00s a.m.
3rd set:				
Direct	23°58'15"		7h21m00s a.m.	
Reversed	23°58'30"		7 23 00 a.m.	
Mean		23°58'22"		7h22m00s a.m.

Greenwich U.C. of Polaris, same date	1h35.4m a.m.
Correction for longitude	- 1.3m
L.M.T. of U.C.	1h34.1m a.m.

	1st set	2nd set	3rd set
Watch time of obsn.	7h11m10s a.m.	7h18m00s a.m.	7h22m00s a.m.
Watch fast L.M.T.	57m40s	57m40s	57m40s
L.M.T of obsn.	6h13m30s a.m.	6h20m20s a.m.	6h24m20s a.m.
	6h 13.5m a.m.	6h 20.3m a.m.	6h 24.3m a.m.
L.M.T. of U.C.	1h 34.1m a.m.	1h 34.1m a.m.	1h 34.1m a.m.
Hour angles, ~~east~~ (west)	4h 39.4m	4h 46.2m	4h 50.2 m

Azimuth of Polaris

Mean time hour angle	Mean declination 89°00'00" Latitude 38°00"	38°31"	40°00'	Correction to azimuth Additive for declination 88°59'54"
4h 39.2m	71.9'	72.4'	74.0'	
4h 39.4m		72.4'		0.1'
4h 46.2m		73.1'		0.1'
4h 49.2m	72.9'	73.4'	75.0'	
4h 50.2m		73.5'		0.1'
4h 59.2m	73.8'	74.3'	75.9'	

Azimuth of Polaris, dec. 89°00'00"	72.4'	73.1'	73.5'
Correction to dec. 88°59'54"(Additive)	0.1'	0.1'	0.1'
	72.5'	73.2'	73.6'
Azimuth of Polaris, dec. 89°59'54"	N. 1°12'30"W.	N 1°13'12"W.	N. 1°13'36"W.
Hor. angle to west of star	23°59'22"	23°58'38"	23°59'22"
True bearing of reference point	N.25°11'52"W.	N.25°11'50"W.	N.25°11'58"W.

Mean true bearing of reference point--- N. 25°11'53" W.

- -

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Date: Dec. 5, 1944	Observer: Hal D. Craig Recorder: M. S. Craig	Instrument, Buff No. 23461.

HOUR ANGLE OBSERVATION OF POLARIS, OBSERVING PROGRAM "b"

Field record

Hour angle observation of Polaris

Telescope	Horizontal angle from monument to Polaris	Watch time, p.m.
Direct	124° 11' 30"	4h 47m 45s
Reversed	124° 11' 30"	4h 49m 20s
Reversed	124° 12' 00"	4h 51m 20s
Direct	124° 12' 00"	4h 52m 30s
Mean --	124° 11' 45"	4h 50m 15s

Watch corrected to 75th meridian standard time,	0m 0s
Correction for longitude -----	-8m 8s
L.M.T. of obs. Dec. 5, 1944,	=4h 42m 07s
Gr. U.C. of Polaris, same date -- 8h 48m 18s, p.m.	
Reduced to long. 77° 01.6' ------------ -51s	=8h 47m 27s p.m.
Hour angle of Polaris, east of meridian	4h 5m 20s
Declination of Polaris,	=89°00'20.4"

Azimuth of Polaris

Mean time hour angles	Mean declination 89° 00' 00" — Latitude 38°	38°53'	40°	Correction subtractive for declination, 89° 00' 20.4"
3h 59.3 m	66.4'	67.2'	68.3'	00.4'
4h 05.3 m		68.2'		00.4'
4h 09.3 m	68.0'	68.8'	69.9'	00.4'

Azimuth of Polaris, 1° 08.2' -00.4'= 1° 07' 48"

Final field notes

December 5, 1944: from a point which is a cross in concrete walk across the street from the south entrance to the South Interior Building, Washington, D.C., I establish my station for observation about 2.00 chs. distant directly on line to the peak of the Washington Monument, in order to clear the vertical angle to Polaris; latitude 38° 53' 30" N., and longitude 77° 01' 36" W.; I correct my watch to the 75th meridian standard time, and at 4h 42m 7s., l.m.t., I make an hour angle observation on Polaris east of the meridian, two each with the telescope in direct and reversed positions, reading the horizontal angle from the peak of the Washington Monument, in the SE. quadrant, to Polaris, east of the meridian.

Mean watch time of obsn. 4h 50m 15s., p.m.

Mean horizontal angle, Polaris to monument,	124° 11' 45"
Azimuth of Polaris ---	1° 07' 48"
Angle from N. to Mon.	125° 19' 33"

True bearing of monument, S. 54° 40' 27" E.

Memo:
Accepted bearing of monument (weighted mean) = S. 54° 40' 25" E.

Example of hour angle observation of Polaris, observing program "b":

Date: October 13, 1944.

Observer: Hugh B. Crawford   Instrument: Buff No. 17993

Field record.

Hour angle observation on Polaris:

Telescope.	Horizontal angle from post to Polaris.	Watch time.
Direct ----	1° 13' 30"	6h 42m 42s
Reversed ---		6 44 22
Direct ----		6 45 22
Reversed ---	4 54 40	6 46 12
Mean	1° 13' 40"	6h 44m 39s

Watch slow of 90th mer. standard war time	+10 40
Correction for longitude ------	-35 12
Correction for war time ----	-1 00 00
L.M.T. of obsn. Oct. 13, 1944 =	5h 20m 07s p.m.

Gr. U.C. of Polaris, Oct. 14, 1944 --- = 12h 17.0m a.m.
Red. to long. 98° 47!9 W. ----= - 1.1

L.M.T. U.C. ------ = 12h 15.9m a.m.
-5 20.1

Hour angle of Polaris E. of the meridian = 6h 55.8m

Declination of Polaris = 89° 00' 01"

Mean time hour angle.	Azimuth of Polaris. Mean declination 89° 00' 01" Latitude		
	38°	39° 07'.4	40°
6h 48.9m	74'.1	75'.3	76'.2
55.8		74 .75	
58.9	73 .3	74 .5	75 .4

Azimuth of Polaris = 1° 14' 45"
" " " by formula = 1° 14' 47"

Final field notes.

October 13, 1944, at a transit point, on a tract known as Outlot Lettered H, situated in the town of Waldo, Russell County, Kansas, in the W. ½ of SW. ¼, sec. 5, T. 11 S., R. 13 W., in latitude 39° 07' 24" N., and longitude 98° 47!.9 W., I find by comparison with a clock in the Union Pacific Depot that my watch is 10m 40s slow of 90th meridian standard war time.

At the same station at 5h 20m 07s p.m., l.m.t., I make an hour angle observation of Polaris, east of the meridian, two each with the telescope in direct and reversed positions, reading the horizontal deflection angle from the West edge of a stone post, 0.6 foot wide, 20.50 chs. N., E. to Polaris.

Watch time of obsn. = 6h 44m 39s p.m.

Mean horizontal angle from Polaris to W. edge of post	= 1° 13' 40" W.
Azimuth of Polaris by inter.	= 1 14 45
Azimuth of Polaris by formula	= 1 14 47
True bearing of W. edge of post	N. 0° 01' 07" E.

Comparison of meridian determined by direct altitude observation of the sun for azimuth, sun south declination:

Mean true bearing of E. edge of post = N. 0° 03' 04" E.
Deflection angle from E. edge of post to W. edge of post = - 0 01 35

Mean true bearing of W. edge of post = N. 0° 01' 29" E.
Mean true bearing of W. edge of post by Polaris obsn. (using azimuth as reduced by formula) = N. 0° 01' 07" E.

1 - 8

HOUR ANGLE OBSERVATION OF POLARIS FOR AZIMUTH AND LATITUDE.

Date: July 7, 1944. Observer: Ty White Instrument: Buff No. 16724

At Curley Seep Guard Station on the Coconino National Forest, in the SE ¼ SW ¼ of sec. 21, T. 24 N., R. 6 E., Gila and Salt River Mer., Arizona, in approximate latitude 35° 26' N., and longitude 111° 46' W., my watch reading correct standard time, as checked by radio time signals, make an hour angle observation of Polaris, east of the meridian, making four observations, two each with the telescope in direct and reversed positions, reading the horizontal angles in the direction N-E to Polaris from a well defined point on a dead pine, approximately 15 chains northwesterly.

To determine the latitude of the station, read the vertical angles to Polaris.

Mean horizontal angle from reference point N-E to Polaris	1° 31' 30"
Mean observed vertical angle	34° 29' 45"
Mean watch time of observation	8h 07m 27s p.m.
Watch fast of l. m. t.	27m 04s
True bearing of reference point	N. 1° 12' 48" W.
Reduced latitude	35° 26' 44" N.

Telescope	Horizontal angle	Vertical angle	Watch time
Direct	1° 31' 30"	34° 31' 00"	8h 03m 00s
Reversed	1 29 00	34 28 00	8 05 45
Reversed	1 30 30	34 28 30	8 08 50
Direct	1 34 00	34 31 30	8 12 12
Mean---	1° 31' 30"	34° 29' 45"	8h 07m 26.7s p.m.

Watch fast of local mean time	- 27m 00.0s
L. M. T. of observation, July 7, 1944	= 7h 40m 26.7s p.m.
Gr. U. C. of Polaris July 8	= 6h 40.8m a.m.
Reduced to longitude 111° 46' W.	= - 1.2
L. M. T. U. C. of Polaris July 8	= 6h 39.6m a.m.
	+12
L. M. T. of observation, July 7	= 7 40.4 p.m.
Hour angle of Polaris east of the meridian	= 10h 59.2m
Declination of Polaris	= 88° 59' 38""

Mean time hour angle	Primary adjustment additive, Polaris below the pole
	Declination
	88° 59' 40"
10h 58.2m	0° 58' 19"
10 59.2	0 58 23
11 10.2	0 59 02

Mean observed vertical angle, v	= 34° 29' 45"
Correction for refraction	= -1 24
h	= 34° 28' 21"
Primary adjustment to elev. of pole	+ 0° 58' 23"
Supplemental corr.	= 0° 00' 00"
Latitude of station	= 35° 26' 44" N.

(See memorandum below)

Mean time hour angle	Azimuth of Polaris			Correction additive for declination 88° 59' 40"
	Mean declination 88° 59' 38"			
	Latitude			
	34°	35° 27'	36°	
10h 58.2m	18'.5	18'.9	19'.0	0'.1
10 59.2		18'.6		
11 08.2	15'.5	15'.8	15'.9	0'.1

Mean horizontal angle from Polaris to reference point =1° 31' 30"

Azimuth of Polaris 18'.6 + 0'.1 = 0° 18' 42" E.

True bearing of reference point = N. 1° 12' 48" W.

Memo:

Elevation of station ≠ 7600 ft.; refraction coefficient = .77

Corrected latitude computation.

Mean observed vertical angle	34° 29' 45"
Refraction (84" x .77 = 65")	- 1 05
Primary adjustment to elevation of pole	+58 23
Supplemental adjustment	0 00
Latitude of station	35° 27' 03" N.

ALTITUDE OBSERVATION OF THE SUN FOR AZIMUTH

Date: Aug. 29, 1944 — Observer: A.W.Brown — Instrument: Buff No 23815
Recorder: R.C.Dice
Timer: W.C.Gale

FINAL FIELD NOTES

Aug. 29, 1944, in camp in sec. 19, T. 45 N., R. 5 E., N.M.P.M., Colo., in latitude 38° 08' N., longitude 106° 21.1' W., with barometer at 22.1 ins. and thermometer approx. 77° F., I make a set of three altitude observations of the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun and reading the horizontal deflection angles from a flag previously set approx. SOUTH of my instrument, to the sun.

Observation	Telescope	Sun	Watch time m.w.t.	Vertical angle	Hor. angle flag to sun
1st	Direct		8h 39m 31s	24° 11' 00"	82° 36' 00"
6th	Reversed		8h 44m 26s	24° 37' 00"	82° 21' 00"
Mean			8h 41m 58.5s	24° 24' 00"	82° 28' 30"
2nd	Direct		8h 40m 29s	24° 22' 00"	82° 27' 00"
5th	Reversed		8h 43m 39s	24° 28' 00"	82° 29' 00"
Mean			8h 42m 04s	24° 25' 00"	82° 28' 00"
3rd	direct		8h 41m 20s	24° 32' 00"	82° 18' 00"
4th	Reversed		8h 42m 41s	24° 18' 00"	82° 39' 00"
Mean			8h42m 0.5s	24° 25' 00"	82° 28' 30"

By 1st observation: flag bears S. 0° 0' 24" E.
By 2nd observation: flag bears S. 0° 0' 02" E.
By 3rd observation: flag bears S. 0° 0' 28" W.

Mean: true bearing of flag = S. 0° 0' 01" W.

TIME CALCULATION

Mean, m.w.t. of observations	8h 42m 1s.
Watch fast of app. time	1h 5m 1s.
Apparent time of observations	7h 37m 0s.

DECLINATION CALCULATION

Sun's decl., Gr. app. noon — 9° 18' 43.6" N.
Hourly difference 53.39"
Red. to long. 106° 21.1'W.: 7h 5m 24s.
Red. to time of obsn. 4h 23m 00s.

2.71h = 2h 42m 24s.
2.71 × 53.39" — 2' 24.7"S.

Decl. of mean sun for period of obsn. 9° 16' 18.9"N

REDUCTION TO TRUE VERTICALS.

	1st Obsn.	2nd Obsn.	3rd Obsn.
v.	24° 24' 00"	24° 25' 00"	24° 25' 00"
Ref.	− 1'30.2"	− 1'30.1"	− 1'30.1"
Parallax	+ 8.0"	+ 8.0"	+ 8.0"
h	24° 22'37.8"	24° 23'37.9"	24° 23'37.9"

AZIMUTH CALCULATIONS

1st observation reduced by the equation:

$$\cos A = \frac{\sin}{\cos \phi \cos h} - \tan \phi \tan h$$

($\delta = 9° 16' 18.9"$; $\phi = 38° 08' 00"$; $h = 24° 22' 37.8"$

log cos ø	9.805741	log sin δ	9.207148	log tan ø	9.894892
" " h	9.959446			" " h	9.656232
	9.855187		9.855187	log	9.551124
		log	9.351961	nat. (−)	.35573
		nat (+)	.22489	(+)	.22489
				cos A (−)	0.13084

A = true bearing of sun = S. 82° 28' 54" E.
Angle from flag to sun = 82° 28' 30"
True bearing of flag = S. 0° 0' 24" E.

2nd Observation Reduced by same Formula:

A = true bearing of sun = S. 82° 28' 02" E.
Angle from flag to sun = 82° 28' 00"
True bearing of flag = S. 0° 0' 02" E.

3rd Observation Reduced by same Formula:

A = true bearing of sun = S. 82° 28' 02" E.
Angle from flag to sun = 82° 28' 30"
True bearing of flag = S. 0° 0' 28" W.

Average of Obsns. 1,2 and 3:

True bearing of flag: S. 0° 0' 1" W.

AZIMUTH CALCULATION

1st Observation Reduced by the Equation:

$$\cos \tfrac{1}{2}A = \sqrt{\frac{\sin S \sin(S - \text{co-decl.})}{\sin \text{co-lat.} \sin \text{co-alt.}}}$$

$90° - \phi = 90° - 38°08'00" = 51° 52' 00"$ = co-lat.
$90° - \delta = 90° - 9°16'18.9" = 80° 43' 41.1"$ = co-decl.
$90° - h = 90° - 24°22'37.8" = 65° 37' 22.2"$ = co-alt.

2 S = 198° 13' 3.3"
S = 99° 06' 31.6"
co-decl. = 80° 43' 41.1"
S − co-decl. = 18° 22' 50.5"

log sin S = 9.994489
log sin S codecl. = 9.498765
" " co-lat. = 9.895741   9.493254
" " co-alt. = 9.959446
9.855187   9.855187
log $\cos^2 \frac{1}{2}A$ = 9.638067
log cos $\frac{1}{2}A$ = 9.819033
$\frac{1}{2}A$ = 48° 45' 33"
A; true bearing of sun = 180° − 97° 31' 6" = S. 82° 28' 54" E.
(Checks calc. by 1st formula)

AZIMUTH CALCULATION

1 st Observation Reduced by the Equation:

$$\tan \tfrac{1}{2}A = \sqrt{\frac{\cos \frac{1}{2}(\zeta + \phi + \delta) \sin \frac{1}{2}(\zeta + \phi - \delta)}{\cos \frac{1}{2}(\zeta - \phi - \delta) \sin \frac{1}{2}(\zeta - \phi + \delta)}}$$

90°
h = 24° 22' 37.8"
ζ = 65° 37' 22.2"
ϕ = 38° 08' 00"
$\zeta + \phi$ = 103° 35' 22.2"
δ = 9° 16' 18.9" (+)
$\zeta + \phi + \delta$ = 113° 01' 41.1"

ζ = 65° 37' 22.2"
ϕ = 38° 08' 00"
$\zeta - \phi$ = 27° 29' 22.2"
δ = 9° 16' 18.9" (+)
$\zeta - \phi + \delta$ = 26° 45' 41.1"

$\frac{1}{2}(\zeta+\phi+\delta) = 56°\ 30'\ 51''$ $\qquad$ $\frac{1}{2}(\zeta-\phi+\delta) = 18°\ 22'\ 51''$

$$\begin{aligned} \zeta+\phi &= 103°\ 45'\ 22.2'' \\ \delta &= \underline{9°\ 16'\ 18.9''}\ (+) \\ \zeta+\phi-\delta &= 94°\ 29'\ 03.3'' \\ \tfrac{1}{2}(\zeta+\phi-\delta) &= \underline{47°\ 14'\ 32''} \end{aligned} \qquad \begin{aligned} \zeta-\phi &= 27°\ 29'\ 22.2'' \\ \delta &= \underline{9°\ 16'\ 18.9''}\ (+) \\ \zeta-\phi-\delta &= 18°\ 13'\ 03.3'' \\ \tfrac{1}{2}(\zeta-\phi-\delta) &= \underline{9°\ 06'\ 32''} \end{aligned}$$

log cos $\frac{1}{2}(\zeta+\phi+\delta)$ =		9.741727
" sin $\frac{1}{2}(\zeta+\phi-\delta)$ =		9.865822
		9.607559
" cos $\frac{1}{2}(\zeta-\phi-\delta)$ =	9.994488	
" sin $\frac{1}{2}(\zeta-\phi+\delta)$ =	9.498768	
	9.493256	(−)9.493256
log $\tan^2 \frac{1}{2}A$ =		0.114303
log $\tan \frac{1}{2}A$ =		0.0571515
$\frac{1}{2}A$ =		48° 45' 33"

A; true bearing of sun 180° − 97° 31' 6" = S. 82° 28' 54" E.

(Checks calc. by 1st and 2nd formulae)

TIME CALCULATION

1st Observation Reduced by the Formula:

$$\tan \tfrac{1}{2}t = \sqrt{\frac{\sin\frac{1}{2}(\zeta+\phi-\delta)\ \sin\frac{1}{2}(\zeta-\phi+\delta)}{\cos\frac{1}{2}(\zeta+\phi+\delta)\ \cos\frac{1}{2}(\zeta-\phi-\delta)}}$$

log sin $\frac{1}{2}(\zeta+\phi-\delta)$ =		9.865822
" " $\frac{1}{2}(\zeta-\phi+\delta)$ =		9.498768
		9.364590
" cos $\frac{1}{2}(\zeta+\phi+\delta)$ =	9.741727	
" " $\frac{1}{2}(\zeta-\phi-\delta)$ =	9.994488	
	9.736215	9.736215
" $\tan^2 \frac{1}{2}t$ =		0.628375
" " $\frac{1}{2}t$ =		9.8141875
$\frac{1}{2}t$ =	33° 06' 2.5"	
t =	66° 12' 5" =	4h 24m 48s

App. time of observation	=	7h 35m 11s.
Equation of time	=	0m 50s.
Local mean time of observation	=	7h 36m 1 s., a.m.
Watch time of observation, m.w.t.	=	8h 41m 58.5s.
Watch fast of l.m.t.		1h 5m 57.5s.

Example of direct altitude observation of the sun for azimuth and time; sun in north declination, and south of an east and west line:

Observer: R. C. Yundt Instrument: Buff and Buff No. 14191

August 4, 1944, at a point 35 links south of the corner of Tps. 7 and 8 N., Rs. 1 and 2 E., Salt Lake Meridian, Utah, in latitude 41°28'40" N., and longitude 111°46'40" W., in order to verify the alignment of my random north boundary, (S. 89°59' W.), I make a series of three altitude observations of the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun, and reading the horizontal angle from a flag on my line to the east, southward to the sun. My watch carries mountain standard war time.

Obsn.	Telescope	Sun	Watch time	Vertical angle	Horizontal angle flag to sun
1st	Direct		10h42m05s a.m.	46°34'00"	21°00'00"
set	Reversed		10 42 59 a.m.	46°10'00"	20°29'00"
	Mean		10 42 32 a.m.	46°22'00"	20°44'30"
2nd	Direct		10h44m02s a.m.	46°54'00"	21°28'00"
set	Reversed		10 44 36 a.m.	46°26'00"	20°49'00"
	Mean		10 44 19 a.m.	46°40'00"	21°08'30"
3rd	Direct		10h45m41s a.m.	47°12'00"	21°52'00"
set	Reversed		10 46 20 a.m.	46°45'00"	21°16'00"
	Mean		10 46 00 a.m.	46°58'30"	21°34'00"

By the 1st observation the flag bears N. 89°59'10" E.
By the 2nd observation the flag bears N. 89°59'23" E.
By the 3rd observation the flag bears N. 89°59'09" E.
Mean true bearing of the flag - - N. 89°59'14" E.

Watch slow M.S.W. time 0m31s

- -

The declination of the sun for the mean period of the three observations is 17°09'30" N.

The following reductions are made to obtain the true vertical angles of the above observations:

	1st obsn.	2nd obsn.	3rd obsn.
v	46°22'00"	46°40'00"	46°58'30"
Refraction	-55"	-55"	-54"
Parallax	+06"	+06"	+06"
h	46°21'11"	46°39'11"	46°57'42"

- -

The following examples of reduction are all by the equation:

$$\text{Cos } A = \frac{\sin\delta}{\cos\phi \cos h} - \tan\phi \tan h$$

1st set:

log cos ø	9.875274	log sin δ	9.469842	log tan ø	9.944941
log cos h	9.838983			log tan h	0.020519
log	9.714257	log	9.714257	log	9.965460
		log	9.755585	nat -	.92355
		nat +	.56962	nat +	.56962
				Cos A -	.35393

True bearing of sun S. 69°16'20" E.
Angle, flag to sun +20°44'30"
S. 90°00'50" E.
True bearing of flag N. 89°59'10" E.

- -

2nd set:

log cos ϕ	9.875274	log sin δ	9.469842	log tan ϕ	9.944941
log cos h	9.836586			log tan h	0.025073
log	9.711860	log	9.711860	log	9.970014
		log	9.757982	nat -	.93328
		nat +	.57277	nat +	.57277
				Cos A -	.36051

True bearing of sun S. 68°52'07" E.
Angle, flag to sun +21°08'30"
S. 90°00'37" E.
True bearing of flag N. 89°59'23" E.

3rd set:

log cos ϕ	9.875274	log sin δ	9.469842	log tan ϕ	9.944941
log cos h	9.834094			log tan h	0.029762
log	9.709368	log	9.709368	log	9.974703
		log	9.760474	nat -	.94342
		nat +	.57607	nat +	.57607
				Cos A -	.36735

True bearing of sun S. 68°26'51" E.
Angle, flag to sun +21°34'00"
S. 90°00'51" E.
True bearing of flag N. 89°59'09" E.

- -

The first set of the above series is selected for an example of reduction by the equation:

$$\text{Cos } \tfrac{1}{2} A = \sqrt{\frac{\text{sin S sin(S-codec)}}{\text{sin colat. sin coalt.}}}$$

90°-ϕ 90°-41°22'40" = 48°37'20" = colat.
90°-δ 90°-17°09'30" = 72°50'30" = codec.
90°-h 90°-46°21'11" = 43°38'49" = coalt.
2 S = 165°06'39"
S = 82°33'20"-
codec. = 72°50'30"
S-codec. = 9°42'50"

log sin S		9.996324
log sin (S-codec)		9.227188
		log 9.223512
log sin colat.	9.875274	
log sin coalt.	9.838983	
log	9.714257	9.714257
log $\cos^2 \frac{1}{2}$ A		9.509255
log cos $\frac{1}{2}$ A		9.754628
$\frac{1}{2}$ A		55°21'49"
A		N. 110°43'38" E.
True bearing of sun		S. 69°16'22" E.
Angle, flag to sun		+ 20°44'30"
True bearing of flag		N. 89°59'08" E.

- -

R. C. Yundt

The first set of the above series is again selected for an example of reduction by the equation:

$$\text{Tan } \tfrac{1}{2} A = \sqrt{\frac{\cos\frac{1}{2}(\zeta+\phi+\delta)\ \sin\frac{1}{2}(\zeta+\phi-\delta)}{\cos\frac{1}{2}(\zeta-\phi-\delta)\ \sin\frac{1}{2}(\zeta-\phi+\delta)}}$$

$$\begin{array}{rl}
 & 90°00'00" \\
h = & \underline{46°21'11"} \\
\zeta = & \underline{43°38'49"} \\
\phi = & \underline{41°22'40"} \\
\zeta+\phi = & \underline{85°01'29"} \\
\delta = & \underline{17°09'30"} \\
\zeta+\phi+\delta = & 102°10'59" \\
\tfrac{1}{2}(\zeta+\phi+\delta) = & 51°05'30"-
\end{array}
\qquad
\begin{array}{rl}
\zeta = & 43°38'49" \\
\phi = & \underline{41°22'40"} \\
\zeta-\phi = & 2°16'09" \\
\delta = & \underline{17°09'30"} \\
\zeta-\phi+\delta = & 19°25'39" \\
\tfrac{1}{2}(\zeta-\phi+\delta) = & 9°42'50"-
\end{array}$$

$$\begin{array}{rl}
\zeta+\phi = & 85°01'29" \\
\delta = & \underline{17°09'30"} \\
\zeta+\phi-\delta = & 67°51'59" \\
\tfrac{1}{2}(\zeta+\phi-\delta) = & 33°56'00"
\end{array}
\qquad
\begin{array}{rl}
\zeta-\phi = & 2°16'09" \\
\delta = & \underline{17°09'30"} \\
\zeta-0-\delta = & -14°53'21" \\
\tfrac{1}{2}(\zeta-\phi-\delta) = & -7°26'41"-
\end{array}$$

$$\begin{array}{lll}
\log\cos\tfrac{1}{2}(\zeta+\phi+\delta) & & 9.798012 \\
\log\sin\tfrac{1}{2}(\zeta+\phi-\delta) & & \underline{9.746812} \\
 & & 9.544824 \\
\log\cos\tfrac{1}{2}(\zeta-\phi-\delta) & 9.996324 & \\
\log\sin\tfrac{1}{2}(\zeta-\phi+\delta) & \underline{9.227188} & 9.544824 \\
 & 9.223512 & \underline{9.223512} \\
\log\tan^2\tfrac{1}{2}A & & 0.321312 \\
\log\tan\tfrac{1}{2}A & & 0.160656 \\
\tfrac{1}{2}A & & 55°21'50" \\
A & \text{N.} & 110°43'40" \text{ E.}
\end{array}$$

True bearing of sun-	S.	69°16'20" E.
Angle, flag to sun		+ 20°44'30"
True bearing of flag	N.	89°59'10" E.

- -

The first set of the above series is likewise selected for an example for the computation for time by the equation:

$$\text{Tan } \tfrac{1}{2} t = \sqrt{\frac{\sin\frac{1}{2}(\zeta+\phi-\delta)\ \sin\frac{1}{2}(\zeta-\phi+\delta)}{\cos\frac{1}{2}(\zeta+\phi+\delta)\ \cos\frac{1}{2}(\zeta-\phi-\delta)}}$$

$$\begin{array}{lll}
\log\sin\tfrac{1}{2}(\zeta+\phi-\delta) & & 9.746812 \\
\log\sin\tfrac{1}{2}(\zeta-\phi+\delta) & & \underline{9.227188} \\
 & & 8.974000 \\
\log\cos\tfrac{1}{2}(\zeta+\phi+\delta) & 9.798012 & \\
\log\cos\tfrac{1}{2}(\zeta-\phi-\delta) & \underline{9.996324} & \\
 & 9.794336 & \underline{9.794336} \\
\log\tan^2\tfrac{1}{2}t & & 9.179664 \\
\log\tan\tfrac{1}{2}t & & 9.589832 \\
\tfrac{1}{2}t & 21°15'03" & \\
t & 42°30'06" = 2h50m00s &
\end{array}$$

Apparent time of observation	9h10m00s a.m.
Equation of time	+ 05m56s
Local mean time of observation	9h15m56s a.m.
Mountain standard war time of observation	10h43m03s a.m.
Watch time of observation	10h42m32s a.m.
Watch slow M.S.W. time	0m31s
Watch fast of l.m.t.	1h26m36s

- -

R. C. Yundt

The same set of the above series is selected for an example for the computation of time by the equation:

$$\operatorname{Sin} t = \frac{\sin A \cos h}{\cos \delta}$$

log sin A	9.970938
log cos h	9.838983
log	9.809921
log cos δ	9.980228
log sin t	9.829693
t = 42°30'04" =	2h50m00s
Apparent time of observation	9h10m00s a.m.
Equation of time	+ 5m56s
Mountain standard war time of obsn.	10h43m02s a.m.
Watch time of observation	10h42m32s a.m.
Watch slow M.S.W. time	0m31s
Watch fast of l.m.t.	$1^h26^m36^s$

- -

MEMO:

The above time equation is convenient when it is required to reduce the observation for azimuth also. However it should not be used when the hour angle approaches 6 hours (when t approaches 90°) as the sine function of an angle approaching 90° changes very slowly.

J - 3

Example of direct altitude observation of the sun for azimuth.

Date: Sept. 20, 1944 Observer: James W. Hardison Instrument: Buff No. 17994

At a point 9.00 chains westerly from the standard ¼ sec. cor., on south boundary of T. 6 N., R. 30 E., Mount Diablo Meridian, Nevada, in latitude 38° 19' 45" N., and longitude 118° 35' W., at 7^h 37^m a.m., app. t., I make a series of four altitude observations upon the sun for azimuth, each with the telescope in direct and reversed positions, observing the sun's center, and reading the horizontal deflection angle from the sun to a reference point to the south.

Observations commenced at 7^h 30^m a.m., and completed at 7 30 a.m., mean 7^h 30^m a.m., by my watch which carries correct 120th meridian standard time.

Obsn.	Telescope	Sun	Vertical angle	Horizontal angle from sun to reference point
1st	Direct	✦	19° 23'	89° 00'
	Reversed		19° 33'	88° 52'
	Mean		19° 28'	88° 56'
2nd	Reversed	✦	19° 36'	88° 48'
	Direct		19° 46'	88° 40'
	Mean		19° 41'	88° 44'
3rd	Direct	✦	19° 51'	88° 35'
	Reversed		20° 02'	88° 27'
	Mean		19° 55½'	88° 31'
4th	Reversed	✦	20° 09'	88° 18'
	Direct		20° 17'	88° 12'
	Mean		20° 13'	88° 15'

By 1st obsn. reference point bears S. 13° 44' 28" W.
By 2nd " " " " S. 13° 44' 24" W.
By 3rd " " " " S. 13° 44' 43" W.
By 4th " " " " S. 13° 44' 49" W.

Mean true bearing of reference point, S. 13° 44' 36" W.

The declination of the sun for the mean period of the four observations = 0° 59' N.

The following reductions are all by the equation

$$\cos A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

log sin. dec.	8.234557		
log cos. lat.	9.894571		
log of quotient	8.339986	log tan lat.	9.897945
log cos. h	9.974550	log tan h	9.547312
	8.365436 = nat. .02320		9.445257

v = 19° 28'
ref.= 2' 42" (-)
par.= 08" (+)
h = 19° 25' 26"

nat. .27878
nat. .02320
cos A = .25558
A = S 75° 11' 32" E.
Hor. ang = 88° 56'
Bearing = S 13° 44' 28" W.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

log of quotient	8.339986	log tan lat	9.897945
log cos. h	9.973965	log tan h	9.552550
	8.366021 = nat. .02323		9.450495
		nat.	.28216
v	= 19° 41'	Nat.	.02323
ref. =	2' 38" (-)	Cos A =	.25893
par. =	08" (+)	A = S.	74° 59' 36" E.
h	= 19° 38' 30"	Hor. ang. =	88° 44'
		Bearing	= S. 13° 44' 24" W.

log of quotient	8.339986	log tan lat	9.897945
log cos. h	9.973307	log tan h	9.558308
	8.366579 = nat. .02326		9.456253
		nat.	.28593
v	= 19° 55' 30"	nat.	.02326
ref. =	2' 38" (-)	cos A =	.26267
par. =	08" (+)	A = S.	74° 46' 17" E.
h	= 19° 53'	Hor. ang.=	88° 31'
		Bearing = S.	13° 44' 43" W.

log of quotient	8.339986	log tan lat	9.897945
log cos. h	9.972499	log tan h	9.565198
	8.367487 = nat. .02331		9.463143
		nat.	.29050
v	= 20° 13'	nat.	.02331
ref. =	2' 35" (-)	cos A =	.26719
par. =	08" (+)	A = S.	74° 30' 11" E.
h	= 20° 10' 33"	Hor. ang =	88° 15'
		Bearing = S.	13° 44' 49" W.

Altitude observation of the sun for azimuth.

This observation is the solar observation commonly used by myself in the field, both for the initiation of a line and for the verification of the azimuth of a line previously run. It differs from the commonly used observing program in certain respects; angles are read to the sun's center, obtained by a careful comparision of the size of that portion of the sun above the upper stadia wire with the corresponding portion of the sun below the lower stadia wire (vertical center), and comparing that portion of each of the above referred to portions of the suns image that appear on each side of the center vertical wire. The resultant determination of the suns center is slightly less accurate in vertical position than in horizontal position, altho not appreciably so. The sighting of the sun's center results in the cross wires of the transit being brightly illuminated regardless of how dark the glass is which is not true when setting tangent to the edges of the sun, and has, for myself, resulted in a more accurate determination of azimuth by the solar method.

Another difference is in sighting with the telescope in reversed position first on alternate observations, reducing the time elapsed in the observation and allowing for a checking of the zero setting of the plate of the transit in the middle of each pair of observations without increasing the elapsed time of the observation to more than 5 or six minutes for a series of eight sighting (four observations.)

As a saving of time when required to reduce the observation in the field (as when initiating a line of survey) the factor sin dec. divided by cos. lat. is first reduced and then is used as a factor common to the entire series of observations.

Example of altitude observation of the sun for azimuth, using morning and afternoon observations:

Date: August 9, 1938. Instrument: Gurley No. 371615

Observer: Arthur D. Kidder.
Recorder: Frank Robertson.

Transcribed field notes.

August 9, 1938, at observation station No. 1, a cross in the sidewalk on the south side of C Street, opposite the south entrance to the Interior Building, Washington, D. C., in latitude 38° 53' 30" N., and longitude 77° 02' W., elevation above sea level 10 ft., and temperature 80° F., at $8^h\ 55^m$ a.m., and $3^h\ 32^m$ p.m., I make series of altitude observations of the sun for azimuth, each series consisting of six observations, three each with the telescope in direct and reversed positions, observing opposite limbs of the sun, and reading the horizontal angle from the tip of the Washington Monument, approximately 40 chs. southeasterly, to the left to the sun in the a.m., and to the right to the sun in the p.m.:

Observation.	Telescope.	Sun.	Apparent time.	Vertical angle.	Horizontal angle monument to sun.
1st	Direct		$8^h\ 54^m\ 00^s$ a.m.	43° 39' 00"	18° 19' 00" to lt.
	Reversed		9 03 50 "	46 00 00	16 55 30 " "
	Mean			44° 49' 30"	17° 37' 15" to lt.
2nd	Direct		$8^h\ 55^m\ 30^s$ a.m.	44° 28' 00"	18° 44' 00" to lt.
	Reversed		9 02 15 "	45 11 00	16 31 00 " "
	Mean			44° 49' 30"	17° 37' 30" to lt.
3rd	Direct		$8^h\ 57^m\ 00^s$ a.m.	44° 12' 00"	17° 41' 30" to lt.
	Reversed		9 00 20 "	45 21 30	17 41 30 " "
	Mean		$8^h\ 58^m\ 49^s$ a.m.	44° 46' 45"	17° 41' 30" to lt.
1st	Direct		$3^h\ 31^m\ 35^s$ p.m.	38° 44' 30"	133° 25' 00" to rt.
	Reversed		3 42 05 "	37 15 00	134 42 00 " "
	Mean			37° 59' 45"	134° 03' 30" to rt.
2nd	Direct		$3^h\ 34^m\ 00^s$ p.m.	38° 58' 00"	133° 01' 00" to rt.
	Reversed		3 40 30 "	37 02 00	135 03 45 " "
	Mean			38° 00' 00"	134° 02' 22" to rt.
3rd	Direct		$3^h\ 35^m\ 15^s$	38° 03' 00"	134° 06' 00" to rt.
	Reversed		3 38 45	37 53 30	134 05 00 " "
	Mean		$3^h\ 37^m\ 02^s$	37° 58' 15"	134° 05' 30" to rt.

By 1st a.m. observation monument bears S. 54° 42' 03" E.
" 2nd " " " " S. 54° 41' 48" E.
" 3rd " " " " S. 54° 41' 00" E.
Mean of a.m. obsns. .... S. 54° 41' 37" E.

By 1st p.m. observation monument bears S. 54° 39' 25" E.
" 2nd " " " " S. 54° 38' 32" E.
" 3rd " " " " S. 54° 39' 50" E.
Mean of p.m. obsns. .... S. 54° 39' 16" E.

Mean, true bearing of monument .... S. 54° 40' 26" E.

Memo:
Accepted bearing of monument (weighted mean) = S. 54° 40' 25" E.

Summary of adjacent [illegible] on the [illegible]

[illegible]

August [illegible]

[illegible]

Field record.

The above observations are reduced for azimuth by the equation:

$$\text{Cos } A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h.$$

1st series, a.m. observations:

Sun's declination, Gr. app. noon, August 9, 1938	15° 58' 32" N.
Red. to long. 77° 02' W., and $8^h 58^m 49^s$ a.m., $2.12^h \times 42"$	1 29 S.
Sun's declination for mean time of observations, δ	15° 57' 03" N.

Latitude, ϕ = 38° 53' 30" N. Refraction coef. = 1.01 x .94 = .95

1st obsn.:

v	=	44° 49' 30"
Refraction (58" x .95 = 55")	=	- 0 55
Parallax	=	+ 6
h	=	44° 48' 41"

Sin δ .... = .274812

Cos ϕ = .778335 tan ϕ = .806658
Cos h = .709431 tan h = .993438

.552175 .552175 -.801365

.497690 .497690

cos A = -.303675
A = S. 72° 19' 18" E.
Hor. ang. = 17 37 15

By 1st a.m. obsn. monument bears	S. 54° 42' 03" E.
" 2nd " " " "	S. 54° 41' 48" E.
" 3rd " " " "	S. 54° 41' 00" E.
Mean	S. 54° 41' 37" E.

2nd series, p.m. observations:

Sun's declination, Gr. app. noon, August 9, 1938	15° 58' 32" N.
Red. to long. 77° 02' W. and $3^h 37^m 02^s$ p.m., $8.75^h \times 42"$	6 08 S.
Sun's declination for mean time of observations, δ	15° 52' 24" N.

Refraction coef. = 1.01 x .94 = .95

By 1st p.m. obsn. monument bears	S. 54° 39' 25" E.
" 2nd " " " "	S. 54° 38' 32" E.
" 3rd " " " "	S. 54° 39' 50" E.
Mean	S. 54° 39' 16" E.
Mean of a.m. and p.m. obsns.	S. 54° 40' 26" E.

J - 5

Date - June 28, 1944 Observer - F. W. Williamson Instrument - W. and L. E. Gurley No. 38105

June 28, 1944, at a point 2 chains South of cor. No. 2, Survey No. 2638, Aniak, Alaska, in latitude 61°34'38" North, longitude 159°42.5' West, at a mean time of 7 hrs. 17 min. P.M., 135° meridian time, I make a series of four (4) altitude observations of the sun for azimuth, each with the telescope in direct and reverse positions, observing opposite limbs of the sun, and reading the horizontal deflection angles from the flag to the sun:

Observation	Telescope	Sun	Time 135° P.M.	Vertical angle	Horizontal angle flag to sun to left
1st	direct		7 h 12 m	23°33'00"	8°12'00"
1st	reverse			23°55'30"	8°29'00"
	mean			23°44'15"	8°20'30"
2nd	reverse		7 h 16 m	23°14'00"	7°36'00"
2nd	direct			23°36'00"	7°53'00"
	mean			23°25'00"	7°44'30"
3rd	direct		7 h 18 m	22°42'30"	6°38'30"
3rd	reverse			23°04'30"	6°58'00"
	mean			22°53'30"	6°48'15"
4th	reverse			22°26'30"	6°10'00"
4th	direct		7 h 22 m	22°46'30"	6°25'30"
	mean			22°36'30"	6°17'45"

Reference point is radio beacon at edge of Aniak Air Field.

By 1st observation beacon bears	N. 76°14'28" W.
By 2nd observation beacon bears	N. 76°15'30" W.
By 3rd observation beacon bears	N. 76°14'33" W.
By 4th observation beacon bears	N. 76°14'17" W.
Mean true bearing of beacon	N. 76°14'42" W.

Field record

The declination of the sun for mean time 7 h 17 m P.M. 135° time for the period of the four (4) observations is 23°14'47" N.

	1st obs.	2nd obs.	3rd obs.	4th obs.
	23°44'15"	23°25'00"	22°53'30"	22°36'30"
ref. and parallax	2'04"	2'06"	2'10"	2'11"
h =	23°42'11"	23°22'54"	22°51'20"	22°34'19"

1st observation reduction by the equation

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin S \sin(S-\text{codecl.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

90° - ø = 90° - 61°34'38"	=	28°25'22"	A.C. log sin 0.322417
90° - decl. = 90° - 23°14'47"	=	66°45'13"	
90° - h = 90° - 23°42'11"	=	66°17'49"	A.C. log sin 0.038275
2S	=	161°28'24"	
S	=	80°44'12"	log sin 9.994299
codecl.	=	66°45'13"	
S - codecl.	=	13°58'59"	log sin 9.383160
		$\cos^2 \frac{1}{2} A$	2) 9.738150
		$\cos \frac{1}{2} A$	9.869075

$\frac{1}{2}$ A = 42°17'29"
2

N.84°34'58" W. true bearing of sun
8°20'30" angle from sun to beacon -
N.76°14'28" W. True bearing to beacon.

Date - June 28, 1944 Observer - F. W. Williamson Instrument - W. and L. E. Gurley No. 38105

2nd Observation

```
   28°25'22"  colat.    A.C. log sin 0.322417
   66°45'13"  codecl.
   66°37'06"  coalt.    A.C. log sin 0.037212
2)161°47'41"
   80°53'50"  S              log sin 9.994496
   66°45'13"
   14°08'37"  S-codecl.      log sin 9.388022
                                  2)9.742147
               cos ½ A   =          9.871072

½ A =    42°00'00"
                 2
N. 84°00'00" W.  true bearing of sun
    7°44'30"         angle from sun to beacon
N. 76°15'30" W.  True bearing to beacon.
```

3rd Observation

```
   28°25'22" colat.    A.C. log sin 0.322417
   66°45'13" codecl.
   67°08'40" coalt.    A.C. log sin 0.035511
2)162°19'15" coal
   81°09'37.5"              log sin 9.994812
   66°45'13"
   14°24'24.5"              log sin 9.395855
                                 2)9.748595
               cos ½ A   =         9.874297

½ A =    41°31'24"
                 2
N. 83°02'48" W.  true bearing of sun
    6°48'15"         angle sun to beacon
N. 76°14'33" W.  True bearing to beacon.
```

4th Observation

```
   28°25'22" colat.    A.C. log sin 0.322417
   66°45'13" codecl.
   67°25'41" coalt.    A.C. log sin 0.034611
2)162°36'16"
   81°18'08"                log sin 9.994977
   66°45'13"
   14°32'55"                log sin 9.400022
                                 2)9.752026
               cos½ A   =          9.876013

½ A =    41°16'01"
                 2
N. 82°32'02" W.  true bearing of sun
    6°17'45"         angle sun to beacon
N. 76°14'17" W.  True bearing to beacon.
```

June 29, 1944 Observer - F. W. Williamson Instrument - W. and L. E. Gurley No. 38105

June 29, 1944, at a point 2 chains South of cor. No. 2, Survey No. 2638, Aniak, Alaska, in latitude 61°34'38" North, longitude 159°42.5' West, at a mean time of 7 hrs. 25 min. A.M., 135° meridian time, I make a series of four (4) altitude observations of the sun for azimuth, each with the telescope in direct and reverse positions, observing opposite limbs of the sun, and reading the horizontal deflection angles from the flag to the sun:

Obser-	Telescope	Sun	Time 135° a.m.	Vertical angle	Horizontal angle flag to sun to right
1st	direct		7 h 20 m	17°55'00"	150°05'00"
1st	reverse			17°35'30"	149°55'00"
	mean			17°45'15"	150°00'00"
2nd	reverse		7 h 22 m	18°15'00"	150°42'00"
2nd	direct			18°04'00"	150°48'30"
	mean			18°09'30"	150°45'15"
3rd	direct		7 h 25 m	18°52'00"	151°52'00"
3rd	reverse			18°31'00"	151°37'00"
	mean			18°41'30"	151°44'30"
4th	reverse			19°18'00"	152°37'00"
4th	direct		7 h 30 m	18°54'00"	152°20'00"
	mean			19°06'00"	152°28'30"

Reference point is radio beacon at edge of Aniak Airfield.

By 1st observation beacon bears	N. 76°13'58" W.
By 2nd observation beacon bears	N. 76°14'45" W.
By 3rd observation beacon bears	N. 76°15'30" W.
By 4th observation beacon bears	N. 76°14'44" W.
Mean true bearing of beacon	N. 76°14'44.5" W.

Field Record.

The declination of the sun for mean time 7 h 25 m A.M. 135° time for the period of the four (4) observations is 23°13'04" N.

	1st obs.	2nd obs.	3rd obs.	4th obs.
	17°45'15"	18°09'30"	18°41'30"	19°06'00"
ref. and parallax	2'52"	2'48"	2'43"	2'38"
	17°42'23"	18°06'42"	18°38'47"	19°03'22"

Observations reduced by the equation

$$\operatorname{Cos} \tfrac{1}{2} A = \sqrt{\frac{\operatorname{Sin} S \sin(S - \text{codecl.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

90° - ø = 90° - 61°34'38" =	28°25'22"	A.C. log sin	0.322417
90° - decl.= 90° - 23°13'04" =	66°46'56"		
90° - h = 90° - 17°42'23" =	72°17'37"	A.C. log sin	0.021077
2S =	167°29'55"		
S =	83°44'57.5"	log sin	9.997411
codecl. =	66°46'56"		
S - codecl. =	16°58'01.5"	log sin	9.465118
		$\cos^2 \frac{1}{2} A$	9.806023
		$\cos \frac{1}{2} A$	9.903011

$\frac{1}{2}$ A = 36°53'01"
2
N. 73°46'02" E. true bearing of sun

150°00'00" angle from sun to beacon
73°46'02"
N. 76°13'58" W. True bearing to beacon 1st obs.

Date - June 29, 1944 Observer - F. W. Williamson Instrument - W. and L. E. Gurley No. 38105

2nd Observation

28°25'22" colat. A.C. log sin 0.322417
66°46'56" codecl.
71°53'18" coalt. A.C. log sin 0.022070
2)167°05'36"
83°32'48" S log sin 9.997240
66°46'56"
16°45'52" S-codecl. log sin 9.460052
2)9.801779
cos ½ A = 9.900889

½ A = 37°15'15"
2
N. 74°30'30" E. true bearing of sun

150°45'15" angle from sun to beacon
74°30'30"
N. 76°14'45" W. True bearing to beacon.

3rd Observation.

28°25'22" colat. A.C. log sin 0.322417
66°46'56" codecl.
71°21'13" coalt. A.C. log sin 0.023416
2)166°33'31"
83°16'45.5" S log sin 9.997005
66°46'56"
16°29'49.5" ½ S-codecl. log sin 9.453267
2)9.796105
9.898052

½ A = 37°44'30"
2
N. 75°29'00" E. true bearing of sun

151°44'30" angle sun to beacon
75°29'00"
N. 76°15'30" W. True bearing to beacon.

4th Observation

28°25'22" colat. A.C. log sin 0.322417
66°46'56" codecl.
70°56'38" coalt. A.C. log sin 0.024477
2)166°08'56"
83°04'28" S log sin 9.996820
66°46'56"
16°17'32" ½S-codecl. log sin 9.447989
2) 9.791703
Cos½ A = 9.895851

½ A = 38°06'53"
2
N. 76°13'46" E. true bearing to sun

152°28'30" angle sun to beacon
76°13'46"
N. 76°14'44" W. True bearing to beacon.

Mean of

Observation taken at this point
June 28th, at 7 h 17 m P.M.
135° time
give course to beacon | N. 76°14'42" W.

Mean of

Observation taken at this point
June 29th at 7 h 25 m A.M.
135° time
give course to beacon | N. 76°14'44.5" W.

Course to beacon mean of
A.M. and P.M. observations | N. 76°14'43" W.

FIELD TEST OF VERTICAL CIRCLE.

In order that the engineer may make proper reductions of astronomical altitude observations when unable to make compensating observations (southeasterly and southwesterly observations at approximately equal altitude), it is essential that he know the the index error, if any, of the vertical circle of his transit throughout the range of vertical angles used in such observations. Normally this range is from 20° to 50°, with the telescope in direct and reversed positions.

There are three methods available for making this vertical circle test in the field:

1. By direct comparison with a transit with known vertical circle indications.

2. By meridian passage observations on a series of equatorial stars throughout this vertical angle range, at a station of known latitude, and comparing the observed vertical angles, corrected for refraction, with the computed true altitude of these stars.

3. By making a series of altitude observations of the sun or a star throughout this angular range, properly balanced by southeasterly and southwesterly observations, at a station of known latitude, and preferably where the meridian has been accurately determined. Then determine the error in the reduced azimuth of each observation, and divide this error by the coefficient taken from the diagram of errors in azimuth due to small errors in vertical angle, to get the indicated error of the vertical angle reading.

The following data are the results of a test of the vertical circle of Buff Transit No. 17993, by the last method, made on March 14 and April 9, 1946, at the G.L.O testing station No. 1, Washington, D. C., in latitude 38° 53' 30" N., and longitude 77° 03' W., orienting on the tip of the Washington Monument, bearing S. 54° 40' 25" E. Each observation in the test consisted of six readings, all with the telescope in the direct or reversed positions, three each on opposite limbs of the sun.

Transit: Buff No. 17993

TELESCOPE DIRECT.

Obs.	Date	Decl.	Vert. angle	Bearing of Monument A. M.	Bearing of Monument P. M.	Hor. error	Coeff.	Vert. error
1	3/14/46	2°38'22"S.	19°15'30"	S.54°40'27"E.		+ 2"	1.02	- 2"
2	"	2°37'51"S.	24°59'15"	S.54°40'07"E.		- 18"	1.16	+ 16"
11	"	2 30 38 S.	24 35 30		S.54°40'32"E.	+ 7	1.15	+ 7
Mean			24°47'22"	S.54°40'20"E.				+ 12
3	"	2°37'28"S.	28°52'50"	S.54°40'17"E.		- 8"	1.32	+ 6"
10	"	2 30 57 S.	27 52 25		S.54°41'02"E.	+ 37	1.26	+ 29
Mean			28 22 37	S.54°40'40"E.				+ 17
4	"	2°36'39"S.	36°44'30"	S.54°39'17"E.		-1'08"	1.93	+ 35"
9	"	2 31 35 S.	34 11 45		S.54°40'53"E.	+ 28	1.66	+ 17
Mean			35 28 07	S.54°40'05"E.				+ 26
5	"	2°36'11"S.	40°15'20"	S.54°40'18"E.		- 7	2.44	+ 3
8	4/9/46	7 35 45 N.	39 59 40		S.54°40'51"E.	+ 26	1.42	+ 18
Mean			40 07 30	S.54°40'34"E.				+ 11
6	3/14/46	2°35'34"S.	44°41'40"	S.54°40'09"E.		- 16	4.00	+ 4
7	"	2 33 09 S.	45 41 30		S.54°40'42"E.	+ 17	4.95	+ 3
Mean			45 11 35	S.54°40'25"E.				+ 3

Mean bearing of monument, telescope direct = S. 54° 40' 25" E.

George W. Johnson,
Ass't Cadastral Engineer,
General Land Office,
Washington, D. C.

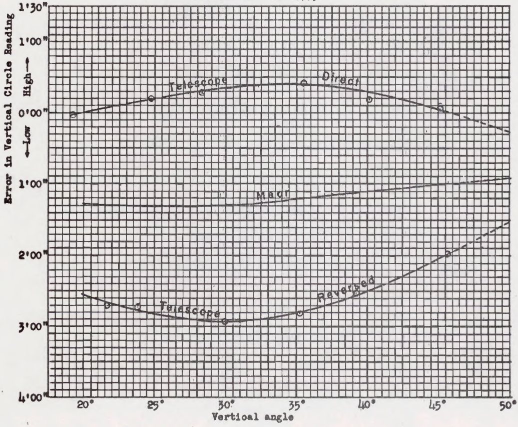
Transit: Buff No. 17993

TELESCOPE REVERSED.

Obs.	Date	Decl.	Vert. angle	Bearing of Monument A. M.	Bearing of Monument P. M.	Hor. error	Coeff.	Vert. error
1	3/14/46	2°38'15"S.	20°36'00"	S.54°43'27"E.		+3'02"	1.05	-2'53"
12	4/9/46	7 36 15 N.	22 47 20		S.54°38'06"E.	-2 19	0.92	-2 31
Mean			21 41 25	S.54°40'46"E.				-2 42
2	3/14/46	2°37'59"S.	23°27'05"	S.54°43'26"E.		+3'01"	1.12	-2'42"
11	4/9/46	7 36 03 N.	24 12 55		S.54°37'49"E.	-2 36	0.95	-2 44
Mean			23 50 00	S.54°40'37"E.				-2 43
3	3/14/46	2°37'23"S.	29°48'32"	S.54°44'40"E.		+4'15"	1.42	-3'00"
10	4/9/46	7 35 35 N.	30 09 00		S.54°37'24"E.	-3 01	1.05	-2 52
Mean			29 58 46	S.54°41'02"E.				-2 56
4	3/14/46	2°36'55"S.	34°18'45"	S.54°45'28"E.		+5'03"	1.68	-3'01"
9	"	2 31 49 S.	36 08 25		S.54°35'32"E.	-4 53	1.86	-2 38
Mean			35 13 35	S.54°40'30"E.				-2 49
5	"	2°36'09"S.	40°46'05"	S.54°46'09"E.		+5'44"	2.55	-2'15"
8	"	2 32 00 S.	37 42 30		S.54°34'41"E.	-5 44	2.04	-2 49
Mean			39 14 17	S.54°40'25"E.				-2 32
6	"	2°35'29"S.	45°05'10"	S.54°49'20"E.		+8'55"	4.50	-1'59"
7	"	2 33 14 S.	46°04'00"		S.54°30'17"E.	-10 08	5.10	-1 59
Mean			45 34 35	S.54°39'49"E.				-1 59

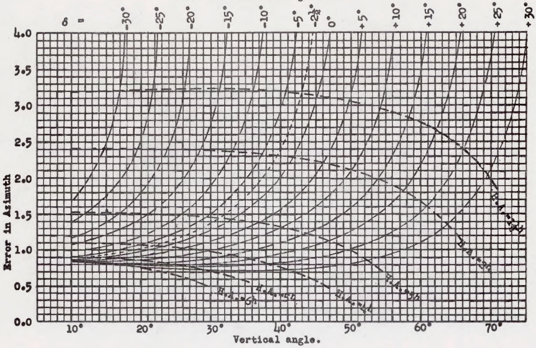
Mean bearing of monument, telescope reversed = S. 54° 40' 31" E.

Vertical Circle Indications.
Transit: Buff No. 17993.



 George W. Johnson

ERRORS IN AZIMUTH RESULTING FROM SMALL ERRORS IN VERTICAL ANGLE
Computed for latitude 40° N.
Error in vertical angle = 1.0



Vertical angle too large = Azimuth of sun or star too small.
" " " small = " " " " " " large.

SUMMARY

The ratio between errors in the reduced azimuth and errors in the vertical angle determination increases as the declination changes southward, and as the vertical angle increases and the hour angle decreases. For example: at vertical angle 30° and declination 10° N. an error of 1' in the vertical angle results in an error of 1' in the reduced azimuth, while at vertical angle 30° and declination 15° S. the same error in vertical angle results in an error of 2.72' (1' 43") in the reduced azimuth.

The precision of results obtained in single latitude observations and altitude observations for azimuth and time are dependent upon the accuracy of the vertical angle determination.

A series of altitude observations in the same quadrant may check within themselves and still be in error if a consistent error is included in the vertical angles (or in the latitude). It therefore becomes important that the vertical circle indications of a transit be well determined.

Such errors in azimuth determination resulting from errors in the vertical angle (as well as errors in latitude) can largely be eliminated by balancing southeasterly observations with southwesterly observations. However, because of the change in the "azimuth error-vertical angle error" ratio at different altitudes and declinations, proper balancing is obtained only when the altitudes and declinations are approximately equal.

Likewise, latitude observations should be balanced by north and south observations; that is, observations on Polaris (or other circumpolar stars) balanced with meridian observations on the sun or stars within the equatorial belt, and preferably at approximately equal altitudes.

Memorandum.

A graph, similar to the above, for latitude 50°, will show much larger errors in azimuth, for the identical vertical angles and declinations; smaller errors in latitudes less than 40°.

G.W.J.

George W. Johnson,
Assistant Cadastral Engineer,
General Land Office,
Washington, D. C.

Example of direct altitude observation of the sun for azimuth, sun south declination:

Date: October 13, 1944.

Observer: Hugh B. Crawford   Instrument: Buff No. 17993

Test of vertical circle, January 5, 1945:

Telescope direct sights low at zero   0°-00' 30"

Telescope reversed sights high at zero   0 05 00

Test of vertical circle, October 14, 1944, against known latitude:

Telescope direct reading reduced to the sun's center crossing the meridian gives plus 0° 00' 36" (reads high)

Telescope reversed reading reduced to the sun's center crossing the meridian gives minus 0° 03' 02" (reads low)

October 13, 1944, at a transit point, on a tract known as Outlot Lettered H, situated in the town of Waldo, Russell County, Kansas, in the W. ½ of SW. ¼, sec. 5, T. 11 S., R. 13 W., in latitude 39° 07' 24" N., and longitude 98° 47' 55" W., at 8:45 a.m., app. t., I make a series of three altitude observations upon the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun; setting first on the East edge of a stone post, 0.6 feet wide, 20.50 chs. N., reversing the telescope, and then reading the horizontal deflection angle SE. to the sun:

Observation.	Telescope.	Sun.	App. time.	Observed Vertical Angle.	Corrected Vertical Angle.	Hor. Angle SE.
1st	Reversed		8h40m00s	24° 30' 00"	24° 33' 00"	55° 42' 00"
"	Direct			24 14 00	24 13 30	56 00 00
	Mean			24° 22' 00"	24° 23' 15"	55° 51' 00"
2nd	Direct		8h45m00s	24° 58' 00"	24° 57' 30"	55° 10' 00"
"	Reversed			24 37 00	24 40 00	55 26 00
	Mean			24° 47' 30"	24° 48' 45"	55° 18' 00"
3rd	Reversed		8h50m00s	25° 23' 00"	25° 26' 00"	54° 33' 00"
"	Direct			25 08 00	25 07 30	54 48 00
	Mean			25° 15' 30"	25° 16' 45"	54° 40' 30"

By 1st obsn. E. edge of post bears N. 0° 03' 04" E.
" 2nd " " " " " " N. 0 03 18 E.
" 3rd " " " " " " N. 0 02 51 E.

Mean true bearing of E. edge of post = N. 0° 03' 04" E.

Field record.

The declination of the sun for the mean period of the three observations = 7° 52' 40" S.

The following reductions are made to obtain the true vertical angles of the above observations:

	1st obsn.	2nd obsn.	3rd obsn.
v =	24° 23' 15"	24° 48' 45"	25° 16' 45"
Refraction =	- 2 07	- 2 04	- 2 02
Parallax =	+ 08	+ 08	+ 08
h =	24° 21' 16"	24° 46' 49"	25° 14' 51"

Reduction to Sun's Center.

Using vertical angle correction values
determined by test by G. W. Johnson.

Obsn.	Telescope.		Bearing of mark.	
1st.	Reversed		N. 0° 03' 19" E.	
"	Direct		N. 0 03 03 E.	
		Mean	N. 0 03 11 E.	N. 0° 03' 11" E.
2nd.	Direct		N. 0° 03' 49" E.	
"	Reversed		N. 0 02 58 E.	
		Mean	N. 0 03 23 E.	N. 0 03 23 E.
3rd.	Reversed		N. 0° 03' 48" E.	
"	Direct		N. 0 02 05 E.	
		Mean	N. 0 02 56 E.	N. 0 02 56 E.
			Mean of all	N. 0° 03' 10" E.

Recapitulation.

Bearing of mark, using vertical angles without eccentricity correction, mean = N. 0° 01' 25" E.
Bearing of mark, using vert. ang. eccentricity corr. determined by meridian observation of sun, mean .... = N. 0° 03' 04" E.
Bearing of mark, using vert. ang. eccentricity corr. determined by test made by G. W. Johnson, mean .... = N. 0° 03' 10" E.
Bearing of auxiallary mark by hour angle obsn. of Polaris = N.0°01'07"E.
Delf. ang. to above mk.= 0 01 35
" N.0°02'42"E. N. 0° 02' 42" E.

K - 1

Date: August 14, 1945 Instrument: Gurley No. 371540

Observer: Oscar B. Walsh
Recorder: Hugh B. Crawford

Example of direct altitude observations on two stars, one in southeast and one in southwest, for azimuth and time, using transit with eccentric vertical circle; telescope equipped with solar circle and double cross wires, spaced 40" apart.

Transcribed Field Notes.

August 14, 1945, at camp in the SE¼SE¼ sec. 15, Township 40 North, Range 14 East, Fourth Principal Meridian, Wisconsin, in latitude 45° 56' 26" N., longitude 88° 43' W., elevation above sea level, approximately 650 ft., temperature 70° F., at $6^h\ 44^m\ 53^s$ p.m., Central Standard time, I make a series of observations on the star α Aquilae (Altair) for azimuth, making four observations, observing simultaneously the vertical angle to the star, and the horizontal angle to a stake, approximately 7 chs. south of my station.

Observation	Telescope	Central Standard time	Vertical angle	Horizontal angle mark to star
1	Direct	$6^h\ 44^m\ 53^s$	32° 15' 00"	66° 47' 30"
2	"	$6^h\ 46^m\ 00^s$	32° 28' 00"	66° 29' 00"
3	Reversed	$6^h\ 48^m\ 00^s$	32° 47' 00"	66° 04' 00"
4	"	$6^h\ 49^m\ 05^s$	32° 56' 00"	65° 51' 00"

By 1st. obsn., mark bears S. 00° 02' 02" E.
" 2nd. " " " S. 00° 02' 22" E.
" 3rd. " " " S. 00° 00' 26" E.
" 4th. " " " S. 00° 00' 47" E.

Mean of southeast observations: S. 00° 01' 24" E.

August 16, 1945, at the same station, in temperature 70° F., at $8^h\ 27^m\ 26^s$ p.m., Central Standard time, I repeat the above observations, observing simultaneously the vertical angle to the star α Virginis (Spica) in the southwest, and the horizontal angle to the same mark, approximately 12 chs. south of my station.

Observation	Telescope	Central Standard time	Vertical angle	Horizontal angle mark to star
1	Direct	$8^h\ 27^m\ 26^s$	21° 37' 30"	45° 30' 00"
2	"	$8^h\ 28^m\ 28^s$	21° 27' 00"	45° 51' 00"
3	Reversed	$8^h\ 32^m\ 54^s$	20° 41' 00"	47° 13' 00"
4	"	$8^h\ 38^m\ 17^s$	19° 45' 00"	48° 50' 00"

By 1st. obsn., mark bears S. 00° 01' 24" W.
" 2nd. " " " S. 00° 00' 19" E.
" 3rd. " " " S. 00° 00' 09" E.
" 4th. " " " S. 00° 00' 22" E.

Mean of southwest observations S. 00° 00' 09" W.
Mean of southeast observations S. 00° 01' 24" E.

Mean, true bearing of mark S. 00° 00' 37" E.

Memo.: Bearing of reference mark by hour angle observation on Polaris, Aug. 2nd, by Mr. Crawford (p. 3): = S.00°00'12"E.

Field Record

The above observations are reduced by the equation:

$$\text{Cos } A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

1st. series, Altair in southeast.

Declination of the star α Aquilae (Altair), August 14, 1945... 8° 43' 30" N.

Latitude 45° 56' 26" N. Longitude 88° 43' 00" W.

Refraction coef. = .99 x .97 = .96

1st. obsn. for azimuth.

v .... = 32° 15' 00"
Refraction (91" x .96) 1' 27"
h .... 32° 13' 33"

Sin δ ....= .1516921
Cos φ = .6954043 tan φ = 1.0333525
Cos h = .8459628 tan h = .6303634

.5882862 .5882862 .6513876

.2578542 .2578542

Cos A = .3935334
A = 66° 49' 32" N.E.
Hor. ang.= 66° 47' 30"

By 1st. obsn., mark bears	S. 00° 02' 02" E.
" 2nd. " " "	S. 00° 02' 22" E.
" 3rd. " " "	S. 00° 00' 26" E.
" 4th. " " "	S. 00° 00' 47" E.

1st. obsn. for time.

$$\text{Cos } t = \frac{\sin h}{\cos \phi \cos \delta} - \tan \phi \tan \delta$$

Sin h .... = .5332577
Cos φ .6954043 tan φ = 1.0333525
Cos δ .9884278 tan δ = .1534681

.6873569 .6873569 .1585866

.7758090 .7758090

Cos t = .6172224
t= 51° 53' 11" =
Sidereal hour angle= $3^h\ 27^m\ 32^s$
Reduced to mean time hour angle= $-\ 34$
Mean time hour angle= $3^h\ 26^m\ 58^s$
Star's transit p.m. $10^h\ 15^m\ 20^s$ l.m.t.
Correct l.m.t. of obsn. $6^h\ 48^m\ 22^s$ p/m.
90th. meridian time($-5^m\ 08^s$) $6^h\ 43^m\ 14^s$ p.m.
Watch time of observation $6^h\ 44^m\ 53^s$ p.m.
Watch fast $1^m\ 39^s$

2nd. series, Spica in southwest, August 16, 1945.

Declination of the star α Virginis (Spica) in southwest, .. - 10° 52' 30"

By 1st. obsn., mark bears	S. 00° 01' 24" W.
" 2nd. " " "	S. 00° 00' 19" E.
" 3rd. " " "	S. 00° 00' 09" E.
" 4th. " " "	S. 00° 00' 22" E.

Example of hour angle observation of Polaris, observing program "b":

Date: August 2, 1945.

Observer: Hugh B. Crawford   Instrument: Gurley No. 371540

Field record.

Hour angle observation of Polaris:

Telescope.	Horizontal angle from post to Polaris.	Watch time.
Direct----	0° 39' 00"	$7^h\ 51^m\ 20^s$
Reversed--		7 52 41
Direct----		7 54 00
Reversed--	2° 33' 30"	7 55 11

Mean 0° 38' 22" $7^h\ 53^m\ 18^s$

Watch fast of l.m.t.-------- -1 5 17

L.M.T. of obsn. Aug. 2, 1945 = $6^h\ 48^m\ 01^s$ p.m.

Gr. U. C. of Polaris, Aug. 3, 1945--------- = $5^h\ 0.5^m$ a.m.
Red. to long. 88° 43' W.----------- = - 1.0
$4^h 59.5^m$ a.m.
L.M.T. of U.C.-------- = 12
L.M.T. of obsn. Aug. 2 = 6 48.0 p.m.

Hour angle, east = $10^h 11.5^m$

August 2, 1945, at a transit point in camp, in the SE. ¼ SE. ¼ sec. 15, T. 40 N., R. 14 E., in approximate latitude 45° 56' 26" N. and longitude 88° 43' 00" W., I find my watch is $1^h\ 5^m\ 17^s$ fast of l.m.t. (~~=~~ Cen. Stan. War T.) by an altitude observation of the sun for time.
At the same station at $6^h\ 48^m\ 01^s$ p.m., l.m.t., I make an hour angle observation of Polaris, east of the meridian, two each with the telescope in direct and reversed position, pointing at a post, south, 7.30 chs. dist., transiting the telescope and reading the horizontal deflection angle east to Polaris.
Watch time of obsn. = $7^h\ 53^m\ 18^s$ p.m.
Mean horizontal angle from Polaris to post = 0° 38' 22"
Azimuth of Polaris by inter. = 0° 38' 10"
Azimuth of Polaris by formula = 0° 38' 11"
S. E.
True bearing of post = 0° 00' 11"
N. 12 W.

Declination of Polaris = +88° 59' 59".2

Mean time hour angle.	Azimuth of Polaris. Mean declination +89° 00' 20" Latitude 44°	45° 56.4'	46°	Correction additive for declination +89°00'00"
10 8.3	37'.7	38' 58"	39'.0	0'.2
11.5		37 58		0 .2
18.3	34 .5	35 40	35 .7	0 .2

Azimuth of Polaris = 0° 38' 10"

Observation for time.	Telescope.	Sun.	Watch time. p.m.	Vertical angle.
	Direct	⊕	$7^h\ 09^m\ 11^s$	13° 06' 00"

Apparent time = $5^h\ 57^m\ 48^s$ p.m.
plus squa. t. = + 6 6
L.M.T. = $6^h\ 03^m\ 54^s$
Watch time = 7 09 11
Watch fast = $1^h\ 5^m\ 17^s$
Decl. of sun for time observation = 17° 40' 10" N.

SUN AND STAR OBSERVATIONS FOR LATITUDE, TIME, AND AZIMUTH

Station: Casa Ybel Hotel, on the south shore of Sanibel Island, Fla.

Latitude:	Scaled from Coast Survey Chart	26° 25' 20" N.
Longitude:	" " " " "	82° 04' 15" W.

Observer, Recorder, and Timer, Jos. C. Thoma

Instrument: Gurley solar transit No. 371536, all circles reading to single minutes.

I set a stake approximately 8 chains northerly from my station for a reference line. Due to the absence of assistants, I did not set the stake exactly on my solar meridian.

March 6, 1945: At 3h20m p.m., app.t., I set off 26° 25' N. on the lat. arc, 5° 32' S. on the decl. arc, and determine a meridian with the solar and find the flag bears N. 0° 02' 30" E.

Polaris Observation for azimuth and latitude

March 6, 1945:

	Watch time	Vert.Ang.	Hor. Ang.
Tel. Dir.	$7^h49^m45^s$ p.m.	27°01'00"	0°57'00" W.
Tel. Rev.	7 56 00 p.m.	27 00 00	0 58 00 W.
Means	7 52 52 p.m.	27 00 30	0 57 30 W.
Watch fast l.m.t.	1 25 05		
L.m.t. of obsn.	6 27 47 p.m.		

L.M.T. U.C. Pol. March 6	2^h 48.9^m p.m.	W. az. of Pol.	0°54'50"
Red. for long.	- .9	Ang.W., flag to star	0°57'30"
	2 48.0 p.m.		
L.m.t. of obsn.	6 27.8 p.m.	Flag bears N.	0°02'40" E.
M.t.h.a. of Pol.	3 39.8.		

The above azimuth of Polaris was obtained by interpolating the tables in the Ephemeris. The following azimuth was obtained by the reduction of the well-known formulae ; using latitude as obtained from a noon observation of the sun on the following day.

M.t. h.a. of Pol.	$3^h39^m48^s$		t = 55°06'00"
Sid. red.	+ 36		φ = 26 24 30
Sidereal h. a.	3 40 24	= 55°06'00"	δ = 89 00 23

Nat. Sin t =		0.82015		
" Cos φ =	0.89565		Nat. Sin. φ =	0.44477
" Tan δ =	57.65850		" cos t =	0.57215
	51.64184			0.25448
	0.25448			
	51.38736	$\frac{51.38736}{0.01597}$ = nat. tan. A.	A = 0°54'50" W.	

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Latitude as above, using tables:

M.t.h.a. Polaris = $3^h\ 39.8^m$ $\qquad \delta = 89°\ 00'\ 23''$

Mean observed vertical angle	=	27° 00' 30"
Correction for refraction	=	- 1 53
h	=	26° 58' 37"
Primary adj. to elev. of pole (Polaris above pole)	=	- 33 46
Supplemental correction	=	- 11
φ	=	26° 24' 40" N.

Latitude as above, using formulae:

$$\cos(\varphi - \alpha) = \frac{\sin\alpha \sin h}{\sin\delta}$$

Sidereal h. a. = $3^h\ 40^m\ 24^s$; t = 55° 06' 00"

$$\text{Nat}\tan\alpha = \frac{\text{nat}\tan\delta\ (57.65850)}{\text{nat}\cos t\ (0.57215)} = 100.77514\ ;\ \alpha = 89°\ 25'\ 53''$$

$$\cos(\varphi - \alpha) = \frac{\sin\alpha\ (.9999507) \times \sin h\ (.4536319)}{\sin\delta\ (.9998496)} = .4536777$$

(φ - α)	=	63° 01' 12"
α	=	89 25 53
φ	=	26° 24' 41" N.

Noon Observation on the Sun for Latitude and Time

March 7, 1945.

	Watch time	Vert.Angle	
Tel. Dir.	$1^h\ 35^m\ 10^s$ p.m.	58° 07' 00"	
Tel. Rev.	1 37 20 p.m.	58 39 00	
Means	1 36 15 p.m.	58° 23' 00"	
		35	Ref.
		05	Par.
		58° 22' 20"	h

Apparent noon	$12^h\ 00^m\ 00^s$		90° 00' 00"
Equation of time	11 10	h	- 58 22 30
L.m.t. of app. noon	12 11 10 p.m.		31 37 30
Watch time of app. noon	1 36 15 p.m.	δ	- 5 12 56 S.
Watch fast l.m.t.	1 25 05	φ	26° 24' 34" N.

Observing the Transit of β Orionis (Rigel), No. 7/11, for latitude and time.

March 7, 1945 Mag. 0.3 δ - 8° 16' long. 82°04'15"W.

Watch time of obsn.	$7^h\ 35^m\ 40^s$ p.m.		
Observed vert. ang.	55° 18' 30"		
Ref.	- 40		
h	55° 17' 50"		

Gr.m.t. of Transit, Mar. 1	$6^h\ 35.1^m$ p.m.		90° 00' 00"
Red. to Mar. 7	- 23.6	h -	55 17 50
Gr.m.t. of transit Mar. 7	6 11.5 p.m.		34 42 10
Red. for long.	.9	δ	8 16 00 S.
L.m.t. transit Mar. 7	6 10.6 p.m.	φ	26° 26' 10" N.

L.m.t. transit Mar. 7	$6^h\ 10^m\ 36^s$ p.m.
Watch t. " " 7	7 35 40 p.m.
Watch fast l.m.t.	1 25 04

Recapitulation

Azimuth of Polaris:

By interpolating the tables	0° 54' 50" W.	Bearing	N. 0° 02' 40" E.
By reducing the formula	0° 54' 50" W.		N. 0° 02' 40" E.
By the solar transit			N. 0° 02' 30" E.

Time

By obsn. on the sun	$1^h\ 25^m\ 05^s$	Watch fast l.m.t.
By obsn. on Rigel	1 25 04	" " "

Latitude:

By obsn. on Polaris, using tables	26° 24' 40" N.
By " " " " formula	26 24 41 "
By noon obsn. on the sun	26 24 34 "
By obsn. on Rigel	26 26 10 "
By scale U.S.C.& G.S. chart	26 25 20 "

Balancing latitude:

Polaris (north) .... 26° 24' 40" N.

Sun	(south)	26° 24' 34" N.	
Rigel	(south)	26 26 10	
	Mean	26° 25' 22" N.	26 25 22

Balanced latitude, mean .... 26° 25' 01" N.

U. S. C. & G. S. chart .... 26° 25' 20" N.

M - 1

Date: June 15, 1945. Instrument: Gurley No. 391237.

Observer: George W. Johnson.
Recorder: Oscar B. Walsh.

Example of direct altitude observation of the sun for azimuth, using transit equipped with solar circle:

Transcribed field notes.

June 15, 1945, at observation station No. 1, a cross in the sidewalk on the south side of C Street, opposite the south entrance to the Interior Building, Washington, D. C., in latitude 38° 53' 30" N., and longitude 77° 02' W., elevation above sea level 10 ft., temperature 70° F., at 8^h 15^m a.m., apparent time, I make a series of altitude observations of the sun for azimuth, making six observations, three each with the telescope in direct and reversed positions, observing simultaneously the vertical angle of the sun's center and the horizontal angle from the tip of the Washington Monument, approximately 40 chs. southeasterly, to the left to the sun:

Observation.	Telescope.	Apparent time.	Vertical angle.	Horizontal angle, monument to sun.
1	Direct	8^h 13^m 25^s	39° 57' 00"	34° 38' 00"
2	"		40 02 00	34 33 00
3	"		40 07 00	34 29 00
4	Reversed		40 20 30	34 18 00
5	"		40 25 30	34 14 00
6	"	8 16 25	40 32 00	34 09 00
Mean		8^h 14^m 55^s		

By 1st obsn. monument bears S. 54° 40' 06" E.
" 2nd " " " S. 54° 41' 01" E.
" 3rd " " " S. 54° 40' 56" E.
" 4th " " " S. 54° 40' 50" E.
" 5th " " " S. 54° 40' 43" E.
" 6th " " " S. 54° 40' 22" E.

Mean of a.m. obsns. ...... S. 54° 40' 40" E.

June 20, 1945, at the same station, in temperature 70° F., at 3^h 42^m p.m., apparent time, I repeat the above observation, observing the horizontal angle from the tip of the Washington Monument to the right to the sun:

Observation.	Telescope.	Apparent time.	Vertical angle.	Horizontal angle, monument to sun.
1	Direct	3^h 42^m 30^s	40° 59' 00"	143° 20' 30"
2	"		40 52 30	143 26 00
3	"		40 45 00	143 32 00
4	Reversed		40 13 00	143 57 00
5	"		40 03 30	144 05 00
6	"	3 50 25	39 27 00	144 34 00
Mean		3^h 46^m 27^s		

By 1st obsn. monument bears S. 54° 41' 09" E.
" 2nd " " " S. 54° 41' 17" E.
" 3rd " " " S. 54° 41' 05" E.
" 4th " " " S. 54° 39' 48" E.
" 5th " " " S. 54° 41' 03" E.
" 6th " " " S. 54° 39' 24" E.

Mean of p.m. obsns. ...... S. 54° 40' 40" E.
Mean of a.m. obsns. ...... S. 54° 40' 40" E.

Mean, true bearing
of monument .... S. 54° 40' 40" E.

Footnote

Observation No. 6, p.m., delayed by a passing cloud.

Memo: Accepted bearing of monument = S. 54° 40' 25" E.

1.

Field record.

The above observations are reduced by the equation:

$$\text{Cos } A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h.$$

1st series, a.m. observations:

Sun's declination, Gr. app. noon, June 15, 1945	23° 18' 35" N.
Red. to long. 77° 02' W., and $8^h\ 14^m 55^s$ a.m., $1.38^h \times 6.5$"	9 N.
Sun's declination for mean time of observations, δ	23° 18' 44" N.

Latitude, ϕ = 38° 53' 30" N. Refraction coef. = 1.01 x .96 = .97

1st. obsn:

v	= 39° 57' 00"
Refraction (69" x .97 = 67") =	- 1 07
Parallax .... =	+ 7
h	= 39° 56' 00"

Sin δ .... = .395741

Cos ϕ = .778335		tan ϕ = .806658
Cos h = .776792		tan h = .837119
.596821	.596821	.675269
	.663082	.663082

cos A = .012187 (−)
A = S. 89° 18' 06" E.
Hor. ang. = 34 38 00

By 1st obsn. monument bears	S. 54° 40' 06" E.
" 2nd " " "	S. 54 41 01 E.
" 3rd " " "	S. 54 40 55 E.
" 4th " " "	S. 54 40 50 E.
" 5th " " "	S. 54° 40 43 E.
" 6th " " "	S. 54 40 22 E.

2nd series, p.m. observations:

Sun's declination, Gr. app. noon, June 20, 1945	23° 26' 25" N.
Red. to long. 77° 02' W., and $3^h\ 46^m\ 27^s$ p.m., $8.9^h \times 1.3$"	12 N.
Sun's declination for mean time of observations, δ	23° 26' 37" N.

Refraction coef. = 1.01 x .96 = .97

By 1st obsn. monument bears	S. 54° 41' 09" E.
" 2nd " " "	S. 54 41 17 E.
" 3rd " " "	S. 54 41 05 E.
" 4th " " "	S. 54 39 48 E.
" 5th " " "	S. 54 41 03 E.
" 6th " " "	S. 54 39 24 E.

The Solar Circle.

The reticle of the transit telescope has the above design, with specifications as follows: Double cross wire at left and bottom (direct position of telescope), spaced 40"; solar circle, radius 15' 45"; stadia ratio 1:132 horizontal and vertical.

Date: June 15, 1945 Instrument: Gurley No. 371540

Observer: Oscar B. Walsh
Recorder: George W. Johnson

Example of direct altitude observation of the sun for azimuth, using transit with eccentric vertical circle; telescope equipped with solar circle.

Transcribed field notes.

June 15, 1945, at observation station No. 1, a cross in the sidewalk on the south side of C Street, opposite the south entrance of the Interior Building, Washington D.C., in latitude 38° 53' 30" N., and longitude 77° 02' W., elevation above sea level, 10 ft., temperature 70° F., at $7^h\ 50^m\ 45^s$ a.m., apparent time, I make a series of observations on the sun for azimuth, making six observations, observing simultaneously the vertical angle to the sun's center and the horizontal angle from the tip of the Washington Monument, approximately 40 chs. southeasterly, to the left to the sun.

Observation	Telescope	Apparent time	Vertical angle	Horizontal angle monument to sun.
1	Direct	$7^h\ 48^m\ 00^s$	33° 14' 00"	39° 52' 00"
2	"		33° 24' 00"	39° 45' 00"
3	"		33° 30' 00"	39° 41' 00"
4	Reversed		33° 57' 00"	39° 20' 30"
5	"		34° 05' 00"	39° 14' 30"
6	"	$7^h\ 53^m\ 30^s$	34° 12' 00"	39° 08' 00"
Mean		$7^h\ 50^m\ 30^s$		

By 1st. obsn., monument bears S. 54° 41' 55" E.
" 2nd. " " " S. 54° 41' 18" E.
" 3rd. " " " S. 54° 40' 47" E.
" 4th. " " " S. 54° 40' 45" E.
" 5th. " " " S. 54° 40' 39" E.
" 6th. " " " S. 54° 41' 50" E.

Mean of a.m. observations ... S. 54° 41' 12" E.

June 20, 1945, at the same station, in temperature 70° F., at $4^h\ 28^m\ 30^s$ p.m., apparent time, I repeat the above observations, observing the horizontal angle from the tip of the Washington Monument to the right to the sun.

Observation	Telescope	Apparent time	Vertical angle	Horizontal angle monument to sun.
1	Direct	$4^h\ 26^m\ 00^s$	34° 42' 30"	148° 17' 00"
2	"		34° 35' 00"	148° 23' 00"
3	"		34° 23' 00"	148° 31' 00"
4	Reversed		33° 57' 00"	148° 51' 30"
5	"		33° 46' 00"	149° 00' 30"
6	"	$4^h\ 31^m\ 30^s$	33° 33' 30"	149° 09' 30"
Mean		$4^h\ 28^m\ 30^s$		

By 1st. obsn., monument bears S. 54° 39' 07" E.
" 2nd. " " " S. 54° 39' 28" E.
" 3rd. " " " S. 54° 38' 13" E.
" 4th. " " " S. 54° 38' 55" E.
" 5th. " " " S. 54° 39' 32" E.
" 6th. " " " S. 54° 39' 06" E.

Mean of p.m. observations S. 54° 39' 03" E.
Mean of a.m. observations S. 54° 41' 12" E.

Mean, true bearing of monument .... S. 54° 40' 08" E.

Memo:
Accepted bearing of monument = S. 54° 40' 25" E.

N - 1

Example of one complete and well-balanced stellar observing program:

(1) The meridian passage of one star for time and latitude;
(2) Polaris for azimuth and latitude; and,
(3) Two well-placed stars in the equatorial belt, one easterly and one westerly, both for time and azimuth.

May 31, 1945 — Instrument: Buff No. 24,847 General Land Office solar transit.

Observations by: Jos. C. Thoma, Cadastral Engineer.
Recording by: Arthur D. Kidder, District Cadastral Engineer; watch reading approximate local mean time.

May 31, 1945, at the testing station, "Troy Cabin", situated on State Highway No. 2, 6 miles east of Troy, New York, and ½ mile east of the Village of Eagle Mills; in latitude 42°44'03"N., and longitude 73°35'15"W.; these values as scaled from the U. S. Geological Survey topographic map, "Troy" quadrangle.

I prepare for one complete stellar observing program. The horizontal angles were turned from a temporary meridian marker, placed at a distance of 24.08 chains north. The mean of a number of previous observations for azimuth indicated that the temporary marker had a bearing of N.00°00'50"E. By observation on the sun at apparent noon today my watch was found to be $0^m\ 24^s$ slow of local mean time.

The following observing program was carried through prior to setting a permanent marker in the true meridian:

For finding position. Star.	Approx. l.m.t., p.m.	Direction of sight.	Vertical Angle.
No. 19/35 α Bootis (Arcturus) 0.2 δ = +19°28'06"	$5^h\ 16^m$	S.88°27'E.	31°07'
No. 13/20 β Geminorum (Pollux) 1.2 δ = +28°09'42"	7 18.5	N.84°20'W.	37°39'
No. 4/ 6 α Urs.Min. (Polaris) 2.2 δ = +89°00'00"	7 30 Sunset	N.00°32'W.	41°48'
No. 17/33 α Virginis (Spica) 1.2 δ = -10°52'36"	8 45.6	South	36°23'

Summary of all results.

	Time. Watch slow of l.m.t.	Latitude.	Azimuth. Indicated bearing of reference mark.
1st equatorial star, SE.	$0^m\ 23^s$		N.0°00'16"E.
2nd " " NW.	0 28		N.0 01 18 E.
Mean			N.0°00'47"E.
Polaris		42°43'44"	N.0 00 57 E.
3rd equatorial star at meridian passage	0 30	42 44 27	
Mean of all	$0^m\ 27^s$	42°44'05"	N.0°00'52"E.
U.S. Geol. Sur. topographic map:		42°44'03"	

1st Star: No. 19/35: α Bootis (Arcturus); δ = +19°28'06"; meridian passage = $9^h\ 36^m.3$ p.m., l.m.t., May 31, 1945; for time and azimuth.

Tel.	Watch time p.m.	Hor. Ang.	Observed Vert. Ang.
Dir.	$5^h\ 19^m\ 05^s$	92°07'30"	31°44'00"
Rev.	5 23 17	92 52 30	32 29 00
"	5 25 15	93 13 00	32 51 30
Dir.	5 28 17	93 45 00	33 26 00
Mean	$5^h\ 23^m\ 58^s$	92°59'30"	v = 32°37'37"

Refraction, r = - 1 30

h = 32°36'07"

S.87°00'14"E. = Reduced azimuth of star.
179°59'44"

N.00°00'16"E. = Indicated bearing of reference mark.

$9^h 36^m 18^s$ = Mer. pass. of star, p.m., l.m.t.
4 11 57 = Reduced mean time hour angle.

$5^h\ 24^m\ 21^s$ $5^h 24^m 21^s$ = Local mean time of observation.

$0^m\ 23^s$ = Watch slow of local mean time.

Apparent noon	$12^h\ 00^m\ 00^s$
Equation of time, subtractive	- 2 29
Local mean time of apparent noon	11 57 31
Watch time of sun's meridian passage	11 57 07
Watch slow of local mean time	24^s

1st Star: No. 19/35:

Star's transit, May 16, meridian of Greenwich = $10^h\ 36.1^m$ p.m.
Reduction to " 31 = $-\ 59.0$
" " long. 73°35'15" = $-\ 0.8$

Meridian passage of star, l.m.t. = $9^h\ 36.3^m$ p.m.

φ = 42°44'03" δ = +19°28'06"   h = 32°36'07"

$$\cos t = \frac{\sin h}{\cos \varphi \cos \delta} - \tan \varphi \tan \delta \qquad \cos A = \frac{\sin \delta}{\cos \varphi \cos h} - \tan \varphi \tan h$$

log	cos	sin	tan	cos	sin	tan
h =		9.731427		9.925536		9.805891
φ =	9.865998		9.965615	9.865998		9.965615
δ =	9.974431		9.548385		9.522817	
	9.840429	9.840429	9.514000	9.791534	9.791534	9.771506
		9.890998			9.731283	
nat		.77803	.32659		.53862	.59089
		.32659				.53862
	cos t =	.45144 (+)		cos A =		.05227 (-)

t = 63°09'50"   A = S.87°00'14"E.

= $4^h 12^m 39^s$ = sidereal hour angle.
$-\ 42$ = reduction to mean time hour angle.
$4^h 11^m 57^s$ = mean time hour angle.

2nd Star: No. 13/20: β Geminorum (Pollux); δ = +28°09'42"; meridian passage = $3^h\ 06.1^m$ p.m., l.m.t., May 31, 1945; for time and azimuth.

Tel.	Watch time p.m.	Hor. Ang.	Observed Vert.Ang.
Dir.	$7^h\ 19^m\ 59^s$	84°06'00"	37°22'00"
"	7 20 53	83 57 00	37 12 00
Rev.	7 23 39	83 31 00	36 40 30
"	7 24 53	83 19 00	36 28 00
Mean	$7^h\ 22^m\ 21^s$	83°43'15"	v = 36°55'37"
			Refraction, r = $-\ 1\ 17$
			h = 36°54'20"

N.83°41'57"W. = Reduced azimuth of star.

N.00°01'18"E. = Indicated bearing of reference mark.

$3^h 06^m 06^s$ = Mer. pass. of star, p.m., l.m.t.
4 16 43 = Reduced mean time hour angle.

$7^h\ 22^m\ 49^s$ $7^h 22^m 49^s$ = Local mean time of observation.

$0^m\ 28^s$ = Watch slow of l.m.t.

Star's transit, May 16, meridian of Greenwich = $4^h\ 05.9^m$ p.m.
Reduction to " 31 = $-\ 59.0$
" " long. 73°35'15" = $-\ 0.8$

Meridian passage of star, l.m.t = $3^h\ 06.1^m$ p.m.

2nd Star: No. 13/20:

$\varphi = 42°44'03''$ $\delta = +28°09'42''$ $h = 36°54'20''$

$$\cos t = \frac{\sin h}{\cos \varphi \cos \delta} - \tan \varphi \tan \delta \qquad \cos A = \frac{\sin \delta}{\cos \varphi \cos h} - \tan \varphi \tan h$$

log	cos	sin	tan	cos	sin	tan
h =		9.778511		9.902887		9.875624
φ =	9.865998		9.965615	9.865998		9.965615
δ =	9.945281		9.728625		9.673906	
	9.811279	9.811279	9.694240	9.768885	9.768885	9.841239
		9.967232			9.905021	
nat		.92733	.49458		.80356	.69381
		.49458			.69381	

cos t = .43275 (+) cos A = .10975 (+)

$t = 64°21'28''$ $A =$ **N.83°41'57"W.**

$= 4^h 17^m 26^s$ = sidereal hour angle.
$\quad - 43$ = reduction to mean time hour angle.
$\underline{\underline{4^h 16^m 43^s}}$ = mean time hour angle.

3rd Star: No. 4/6: α Urs. Min. (Polaris); $\delta = +89°00'00''$; meridian passage at upper culmination = $9^h\ 10^m\ 00^s$ a.m., l.m.t., May 31, 1945; sunset observation for azimuth and latitude.

Tel.	Watch time p.m.	Hor.Ang. Repeated		Observed Vert.Ang.
Dir.	$7^h\ 28^m\ 03^s$	0°34'30"		41°50'30"
"	7 30 07			41 49 00
"	7 32 35	1°41'30"		41 49 00
Mean	$7^h\ 30^m\ 15^s$	0°33'50"	(1/3)	41°49'30"
Rev.	$7^h\ 35^m\ 01^s$	0°33'00"		41°50'00"
"	7 36 33			41 50 00
"	7 38 07	1°38'00"		41 50 00
Mean	$7^h\ 36^m\ 34^s$	0°32'40"	(1/3)	41°50'00"
	$7^h\ 33^m\ 24^s$	0°33'15"	Mean of all.	v = 41°49'45"
			Refraction, r =	− 1 04
	0 27	Watch slow of l.m.t.		h = 41°48'41"
	$7^h\ 33^m\ 51^s$	p.m., l.m.t. of observation.		
	12			
U.C.	$9^h\ 10^m\ 00^s$	a.m.		
	$10^h\ 23^m\ 51^s$	= Hour angle of Polaris, west of meridian.		

3rd Star: Polaris:

Vertical angle, corrected for refraction:	h = 41°48'41"
Vertical angle correction to elevation of pole for hour angle 10h 23.85m, additive:	= + 0°55'03"
Latitude:	= 42°43'44"
Observed horizontal angle:	= 0°33'15"
Azimuth of Polaris for hour angle 10h 23.85m west of the meridian:	A = N.0°32'18"W.

Indicated bearing of reference mark:		N.0°00'57"E.
Same, 1st equatorial star, SE.:	N.0°00'16"E.	
" 2nd " ", NW.:	N.0°01'18"E.	
Mean of the equatorial stars:	N.0°00'47"E.	N.0°00'47"E.
Determined true bearing of reference mark:		N.0°00'52"E.

Upper culmination, May 31, meridian of Greenwich:	$9^h 10.8^m$ a.m.
Reduction to long. 73°35'15"	= - 0.8
	$9^h 10.0^m$ a.m.

Interpolation from tables in the Ephemeris:

For azimuth.

Hour Angle.	Latitude. 42°00'	42°44'	44°00'
10^h 18.3^m	33.5		34.5
22.3			
23.85	31.7	32.1	32.7
28.3	30.3		31.3
34.3			

For latitude.

Hour Angle.	Declination. 88°59'50"	89°00'00"	89°00'10"
22.3	0°55'03"	0°54'54"	0°54'45"
23.85		0 55 03	
34.3	0 56 14	0 56 05	0 55 56

For δ = 89°00'00" + 0.2

32.3

A = N.0°32'18"W.

Primary adjustment to elevation of pole, additive; no supplemental correction: 0°55'03"

4th Star: No. 17/33: α Virginis (Spica); δ = -10°52'36"; meridian passage = $8^h\ 45.6^m$ p.m., l.m.t., May 31, 1945; for time and latitude.

Star's transit, May 16, meridian of Greenwich	=	$9^h\ 45.4^m$ p.m.
Reduction to " 31	=	- 59.0
" " long. 73°35'15"	=	- 0.8
Meridian passage of star, l.m.t.	=	$8^h\ 45.6^m$ p.m.
	=	$8^h 45^m 36^s$ p.m.

Tel.	Observed Vert.Ang.	Watch time p.m.			
Dir.	36°24'00"	$8^h\ 45^m\ 06^s$	Watch time	=	8 45 06
Rev.	36 24 30				
			Watch slow of l.m.t.	=	$0^m 30^s$
Mean	36°24'15"		Same, 1st star	=	0 23
	- 1 18	= refraction.	" 2nd "	=	0 28
	36°22'57" = h		Mean	=	$0^m 27^s$
δ =	10 52 36 S.				
	47°15'33"				
φ =	42 44 27	= Latitude of station		=	42°44'27"
	90°00'00"	Same, by Polaris observation		=	42 43 44
			Mean		42°44'05"

N - 2

Stellar Hour Angle Observation for Azimuth and Time.

Observer: Geo. F. Rigby

Recorder: Norman D. Price

Date: October 26, 1944. Instrument: W. & L. E. Gurley No. 2350

October 26, 1944, in the SW¼NW¼ of sec. 25, T. 1 N., R. 1 E., Willamette Meridian, Oregon, latitude 45° 32' 30" N., and longitude 122° 39' 00" W., at 4^h 00^m p. m., app. t., or 4^h 54.6m p. m., by my watch which reads correct Pacific War Time, I set off 45° 32' 30" N., on the lat. arc; 12° 36' 15" S., on the decl. arc; and determine a meridian with the solar, setting a nail on the meridian thus determined in a hub 10 chs. N. of my station. After dark, to test this indication of the solar and the reading of my watch, I make a series of six altitude observations of the star alpha Ophiuchi for azimuth and time, three each with the telescope in direct and reversed positions, reading the horizontal deflection angles from hub to star.

Star: 22/44 2.1

Alpha Ophiuchi

+12° 36.2'

Obs'n.	Telescope	Watch Time	Vertical Angle	Horizontal Angle from hub to star
1	Direct	6^h 50^m 05^s	44° 33' 30" r -0 59 44° 32' 31"	55° 32' 00" SW.
2	Direct	6^h 51^m 16^s	44° 24' 00" r -0 59 44° 23' 01"	55° 50' 30" SW.
3	Direct	6^h 52^m 21^s	44° 13' 00" r -1 00 44° 12' 00"	56° 11' 00" SW.
4	Reversed	6^h 54^m 59^s	43° 52' 00" r -1 00 43° 51' 00"	56° 50' 00" SW.
5	Reversed	6^h 58^m 55^s	43° 17' 30" r -1 01 43° 16' 29"	57° 53' 30" SW.
6	Reversed	7^h 00^m 13^s	43° 05' 30" r -1 02 43° 04' 28"	58° 14' 30" SW.

r = refraction in zenith distance.

Stellar Hour Angle Observation for Azimuth and Time.

Azimuth

$$\cos A = \frac{\sin d}{\cos \phi \cos h} - \tan \phi \tan h$$

A = Star's azimuth
d = Declination
h = Altitude corrected for ref.
ϕ = Latitude

log sin d	9.338855					
log cos ϕ	9.845341					
diff.	9.493514	9.493514	9.493514	9.493514	9.493514	9.493514
log cos h	9.852930	9.854107	9.855465	9.858029	9.862177	9.863601
diff.	9.640584	9.639407	9.638049	9.635485	9.631337	9.629913
1st term	.43710	.43592	.43456	.43200	.42790	.42649
log tan ϕ	.008212	.008212	.008212	.008212	.008212	.008212
log tan h	9.993056	9.990655	9.987871	9.982562	9.973829	9.970787
sum	.001268	9.998867	9.996083	9.990774	9.982041	9.978999
2nd term	1.00292	.99740	.99102	.97898	.95949	.95279
nat. cos A	.56582	.56148	.55646	.54698	.53159	.52630
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
angle A	55 32 28	55 50 30	56 11 20	56 50 24	57 53 12	58 14 38
hor. angle	55 32 00	55 50 30	56 11 00	56 50 00	57 53 30	58 14 30

azimuth	S. 0° 00' 28" W.	Mean azimuth S. 0° 00' 10" W.
"	South	
"	S. 0° 00' 20" W.	
"	S. 0° 00' 24" W.	
"	S. 0° 00' 18" E.	
"	S. 0° 00' 08" W.	

Time.

$$\cos t = \frac{\sin h}{\cos \phi \cos d} - \tan \phi \tan d$$

t = Star's hour angle
d = Declination
h = Altitude corrected for ref.
ϕ = Latitude

log sin h	9.845985	9.844762	9.843336	9.840591	9.836006	9.834388
log sin ϕ	9.845341	9.845341	9.845341	9.845341	9.845341	9.845341
diff.	.000644	9.999421	9.997995	9.995250	9.990665	9.989047
log cos d	9.989407	9.989407	9.989407	9.989407	9.989407	9.989407
diff.	.011237	.010014	.008588	.005843	.001258	9.999640
1st term	1.02621	1.02330	1.01997	1.01355	1.00290	.99917
log tan ϕ	.008212					
log tan d	9.349448					
sum	9.357660					
2nd term	.22786	.22786	.22786	.22786	.22786	.22786
nat. cos t	.79835	.79544	.79211	.78569	.77504	.77131
	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
t	37 01 39	37 18 11	37 37 00	38 12 57	39 11 28	39 31 41
	h m s	h m s	h m s	h m s	h m s	h m s
red. to time	2 28 07	2 29 13	2 30 28	2 32 52	2 36 46	2 38 07
red. to m.t.	-25	-25	-25	-25	-26	-26
l. m. t.	2 27 42	2 28 48	2 30 03	2 32 27	2 36 20	2 37 41
transit t.	3 11 34	3 11 34	3 11 34	3 11 34	3 11 34	3 11 34
l.m.t. obsn	5 39 16	5 40 22	5 41 37	5 44 01	5 47 54	5 49 15
red. P.W.T.	1 10 36	1 10 36	1 10 36	1 10 36	1 10 36	1 10 36
P.W.T. obsn	6 49 52	6 50 58	6 52 13	6 54 37	6 58 30	6 59 51
watch time	6 50 05	6 51 16	6 52 21	6 54 59	6 58 55	7 00 13
watch fast	0 00 13	0 00 18	0 00 08	0 00 22	0 00 25	0 00 22

Mean: watch 18 seconds fast of Pacific War Time

O - 1

DIRECT ALTITUDE OBSERVATION UPON THE SUN FOR AZIMUTH, IN ALASKA

Date : September 9,1945. Observer: B.J.Kinsey. Instrument: Buff No. 18,000.

At my station 10 chs. east of the ½ sec. cor. of secs. 13 and 18, T.3 N.,R.11 W., Seward Base and Meridian, Alaska, in latitude 60°21'N. and longitude 151°20'W., I make a series of four observations upon the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun, and reading the deflection angle from a flagpole about 10 chs. to the S., SW to the sun:

Observation	Telescope	Sun	Watch time Std.War	Vertical angle	Horizontal angle flag to sun	
1st. set	Direct		5h09m	18°46'	65°32'	to SW.
" "	Reversed		5h11m	19°09'	66°21'	" "
	Mean		5h10m	18°57'30"	65°56'30"	" "
2nd. "	Direct		5h13m	18°31'	66°35'	" "
" "	Reversed		5h15m	18°56'	66°15'	" "
	Mean		5h14m	18°43'30"	66°25'	" "
3rd. "	Direct		5h16m	18°16'	66°32'	" "
" "	Reversed		5h18m	18°41'	67°20'	" "
	Mean		5h17m	18°28'30"	66°56'	" "
4th. "	Direct		5h20m	18°01'	67°36'	" "
" "	Reversed		5h21m	18°28'	67°14'	" "
	Mean		5h20m30s	18°14'30"	67°25'	" "

The declination of the sun for the mean period of the four observations = 5°08' N.

The following reductions are accomplished to obtain the true vertical angles of the above observations:

		1st. obsn.	2nd. obsn.	3rd. obsn.	4th. obsn.
v	=	18°57'30"	18°43'30"	18°28'30"	18°14'30"
Refraction	= -	2'46"	2'48"	2'51"	2'54"
Parallax	= +	8"	8"	8"	8"
h	=	18°54'52"	18°40'50"	18°25'47"	18°11'44"

The reduction of the above series is accomplished by the equation:

$$\text{Cos A} = \frac{\pm \text{ Sin D}}{\text{Co-Sin L Co-Sin H}} - \text{Tan L Tan H}$$

1st.set	Log Sin D	=	8.9516957		Log Tan L	=	0.2447092
	" Cos L	=	9.6943423		" " H	=	9.5348611
			9.2573534		"2nd.term	=	9.7795703
	" " H	=	9.9758929				
	"1st.term	=	9.2814605	.60196	" " "		
	" " "	=		.19119			
	Nat.Cos A	=		.41077			
	Angle A	=	65°45'				
2nd. set	Log Sin D	=	8.9516957		Log Tan L	=	0.2447092
	" Cos L	=	9.6943423		" " H	=	9.5290145
			9.2573534		"2nd.term	=	9.7737237
	" " H	=	9.9764963				
	"1st.term	=	9.2808571	.59396	" " "		
	" " "	=		.19098			
	Nat. Cos A	=		.40304			
	Angle A	=	66°14'				
3rd. Set	Log Sin D	=	8.9516957		Log Tan L	=	0.2447092
	" Cos L	=	9.6943423		" " H	=	9.5227466
			9.2573534		"2nd.term	=	9.7674558
	" " H	=	9.9771345				
	"1st.term	=	9.2802189	.58540	" " "		
	" " "	=		.19065			
	Nat. Cos A	=		.39475			
	Angle A	=	66°45'				
4th. set	Log Sin D	=	8.9516957		Log Tan L	=	0.2447092
	" Cos L	=	9.6943423		" " H	=	9.5167962
			9.2573534		"2nd.term	=	9.7615054
	" " H	=	9.9777219				
	"1st.term	=	9.2796315	.57743	" " "		
	" " "	=		.19038			
	Nat Cos A	=		.38705			
	Angle A	=	67°14'				

Bearing of flagpole:

1st. obsn.	:	S. 0°11'30" E.
2nd. "	:	S. 0°11'00" E.
3rd. "	:	S. 0°11'00" E.
4th. "	:	S. 0°11'00" E.
Mean	:	S. 0°11'07" E.

STELLAR OBSERVATION FOR MERIDIAN IN ALASKA

Date: September 9,1945. Observer: B.J.Kinsey Instrument: Buff 18,000.

Star: 19/35 0.2 δ = 19° 28.1' N.
Bootis (Arcturus)

Gr.m.t. stars transit, September 1	:	3h31m p.m.
Red. to " 9	:	- 31.5m
" " long. 151°20'W.	:	- 1.6m
Stars transit l.m.t. " "	:	2h57.9m
Watch fast of l.m.t.	:	0h59m13s

At my station 10 chs. east of the ½ sec. cor. of secs. 13 and 18, T.3 N., R.11 W., Seward Base and Meridian, Alaska, in latitude 60°21'N. and longitude 151°20'W., I make four altitude observations upon the star Arcturus for azimuth, two each with the telescope in direct and reversed positions and reading the horizontal deflection from a flagpole set about 10 chs. S.0°11'E., as determined by previous altitude observations upon the sun for azimuth.

Position of Telescope		Time: Std. watch	Horizontal angle	Vertical angle
1st. set	Direct :	7h14m p.m.	93°50'00"	20°30'00"
" "	Reversed :	7h15m "	94°12'00"	20°18'00"
	Mean :	7h14m30s p.m.	94°01'00"	20°24'00"
			Refraction :	2'34"
			True vertical angle :	20°21'26"
2nd. set	Direct :	7h16m p.m.	94°29'00"	20°06'00"
" "	Reversed :	7h17m "	94°45'00"	20°00'00"
	Mean :	7h16m30" p.m.	94°37'00"	20°03'00"
			Refraction :	2'37"
			True vertical angle :	20°00'23"

The reduction of the above series is accomplished by the equation:

$$\text{Cos A} = \frac{\pm \text{ Sin D}}{\text{Cos L Cos H}} - \text{Tan L Tan H}$$

1st. set

Log Sin D =	9.5227811	Log Tan L =	0.2447092
" Cos L =	9.6943423	" " H =	9.5694289
	9.8284388	" 2nd.term=	9.8141381
" " H =	9.9719907		
" 1st.term =	9.8564481		
" " " =	.71853		
	.65184	" " "	
Nat. Cos A =	.06669 NW.		
Angle A =	86°10'30"(star to ~~NE~~).		

Bearing of reference point by this observation : S.0°11'30"E.

2nd. set

Log Sin D =	9.5227811	Log Tan L =	0.2447092
" Cos L =	9.6943423	" " H =	9.5612165
	9.8284388	"2nd.term =	9.8059257
" " H =	9.9729682		
"1st.term =	9.8554706		
" " " =	.71692		
	.63962	" " "	
Net Cos A =	.07730 NW.		
Angle A =	85°34' (star to ~~NE~~)		

Bearing of reference point by this observation : S.0°11'E.

Memo:

Azimuth by obsn. on Arcturus (westerly), mean	= S. 0° 11' 15" E.
" " " " Sun (southwest), mean	= S. 0 11 07 E.
Mean	= S. 0° 11' 11" E.
" " " " Polaris (north)	= S. 0 11 00 E.
Balanced azimuth	= S. 0° 11' 05" E.

OBSERVATION UPON POLARIS AT ELONGATION
(ALASKA)

Date: September 9,1945. Observer: B.J.Kinsey. Instrument: Buff 18,000.

September 9,1945, Gr. E.E. of Polaris, Lat. 60°21' N.	:	8h 36.0m, p.m.
Red. to long. 151°20'W.	:-	1.6m
" " lat. 60°21'N.	:+	3.7m
L.M.T. of E.E. of Polaris	:	8h 38.1m p.m.

Sept. 9,1945:

At my station 10 chs. east of the ¼ sec. cor. of secs. 13 and 18, T.3 N., R.11 W., Seward Base and Meridian, Alaska, in latitude 60°21'N. and longitude 151°20'W., at 8h38m., p.m.,l.m.t. I observe Polaris at eastern elongation,making two observations, one each with telescope in direct and reversed positions, deflecting angles from a reference point established previously by direct observation upon the sun for azimuth, the solar bearing of which is S.0°11'E.

Telescope		Time Watch:Std. War.	Horizontal angle reference pt.to star.	Vertical angle.
Direct	:	9h 37m., p.m.	N. 2° 12' E.	60° 20' 30"
Reversed	:	9h 39m., "	N. 2° 11' E.	60° 21' 30"
Mean	:	9h 38m "	N. 2° 11'30"E.	60° 21' 00"

Azimuth of Polaris at E.E.	:	2° 00' 59".
Declination of Polaris	:	89° 00' 08".
Watch fast of l.m.t.	:	0h 59m 13s

The bearing of the reference point by Polaris obsn. is therefore S.0°10'31"E.

OBSERVATION UPON THE SUN FOR TIME AND LATITUDE
(ALASKA)

Date: September 10,1945.

At same station as described above, I make a meridian obsn. of the sun for time and latitude, observing simultaneously the altitude of the suns lower limb and the transit of the suns west limb, reversing the telescope and observing simultaneously the altitude of the suns upper limb and the transit of the suns east limb:

Telescope		Sun	Time: Std. War.	Altitude
Direct	:	♃	12h 55m 10s p.m.	34° 14'
Reversed	:	♄	12h 57m 20s "	34° 45'
Mean	:		12h 56m 15s "	34° 29'30"
			Refraction: -	1'22"
				34° 28'08"
			Parallax : +	07"
			True altitude:	34° 28'15"

Watch time of apparent noon	=	12h 56m15s
Apparent noon	=	12h 00m00s
Equation of time	=	2m58s
L.m.t. of apparent noon	=	11h57m02s
Watch fast of l.m.t.	=	59m13s

Suns declination, Gr.noon	=		4° 58' 43" N.
Red. to long. 151°20'W.	=	or 10h 5m	
" " time of obsn.	=	56.86s x 10 hrs.x 56.86(568")=	9' 29"
Declination of the sun at apparent noon:			4° 49' 14"
			+90° 00' 00"
			94° 49' 14"
			- 34° 28' 08"
Latitude of station	=		60° 21' 06"

Polaris at sunrise and sunset; the sun at
meridian passage; time, latitude, and azimuth.

Camp Case, Mohican Forest Park, Loudonville, Ohio.	Latitude: 40°36'22"N.	Longitude: 82°18'33"W.
July 31, 1940.	Arthur D. Kidder, observing; watch reading approximate local mean time.	Buff No. 10898 General Land Office solar transit.

Horizontal angles at my station at the camp, reading to a flag pole on a building which by observation I find to bear N.25°13'16"E., 1½ Mi. dist.

Tel.	Watch Time a.m.	Hor.Ang. NE-N			Observed Vert.Ang.
			U.C., Gr., July 31:	$5^h09.0^m$ a.m.	
			Red. for long.	- 0.9	
Rev.	$4^h 36^m 30^s$	25°02'00"	U.C., l.m.t.:	$5^h08.1^m$ a.m.	41°39'00"
Dir.	4 46 30	25 06 00			41 38 30
Mean	$4^h 41^m 30^s$	25°04'00"	Decl. = 88°58'33"N.		v = 41°38'45"
Watch					r = - 1 05
slow of l.m.t.	$1^m 00^s$				h = 41°37'40"
			Subtract to elevation of pole		= 1 1 03
L.M.T.	$4^h 42^m 30^s$				
U.C.	5 08 06	0°09'12" = Azimuth		Latitude	= 40°36'37"
Hour Ang. east	$0^h 25^m 36^s$	N.25°13'12"E. = Indicated bearing to flag pole.			
	p.m.		U.C., Gr., Aug. 1:	$5^h05.1^m$ a.m.	
			Red. for long.	- 0.9	
Dir.	$7^h 15^m 35^s$	24°30'30"			39°46'00"
Rev.	7 18 00	24 29 00	U.C., l.m.t.:	$5^h04.2^m$ a.m.	39 47 00
Mean	$7^h 16^m 48^s$	24°29'45"			v = 39°46'30"
Watch					r = - 1 10
slow of l.m.t.	$1^m 00^s$				h = 39°45'20"
			Add to elevation of pole		= 0 51 44
L.M.T.	$7^h 17^m 48^s$				
U.C., Aug. 1 a.m.	12 5 04 12	0°43'36" = Azimuth		Latitude	= 40°37'04"
		N.25°13'21"E. = Flag pole.		Mean latitude	= 40°36'50"
Hour Ang. east	$9^h 46^m 24^s$	N.25°13'16"E. = Mean indicated bearing to flag pole.			

Tel.	Sun	Watch Time			Observed Vert.Ang.
			Apparent noon	$12^h 00^m 00^s$	
			Equation of time	+ 6 14	
Rev.		$12^h 04^m 08^s$	App. noon, l.m.t.	$12^h 06^m 14^s$	67°20'30"
		+ 1 06	Add to time for sun's center.		
		$12^h 05^m 14^s$			
Dir.					67°51'30"
L.M.T.		$12^h 06^m 14^s$	Decl. =	18°11'15"N.	v = 67°36'00"
				90 00 00	r = - 24
Watch slow		$1^m 00^s$			p = + 04
				108°11'15"	
				67 35 40	h = 67°35'40"
				40°35'35"	= Latitude
Mean of two observations on Polaris				= 40 36 50	
Balanced mean latitude				= 40°36'12"	
Topographic map U.S.G.S.				= 40°36'22"	

For demonstration of the General Land Office solar transit, meeting of The Society for the Promotion of Engineering Education, Civil Engineering Division, Surveying and Geodesy.

[illegible]

P - 1

EQUAL ALTITUDE OBSERVATION OF THE SUN

Observer: John S. Knowles
Instrument: Buff solar transit No. 23819
Watch: Illinois- Bunn Special

September 2, 1944, near the corner of secs. 5, 6, 31, and 32, on the south boundary of T. 13 S., R. 90 W., ~~New Mexico~~ 6th Prin. Mer. Principal Meridian, Colorado, in latitude 38° 52'.4 N. and longitude 107° 29' W., at 8h 34m a.m. and 3h 30m p.m. app. t., I make a series of three altitude observations upon the sun for azimuth, reading the horizontal deflection angle from a nail driven firmly in a post about 6 chs. NNE., clockwise in the a.m, to the sun's right limb, and counter-clockwise in the p.m. to the sun's left limb; equal vertical angles being taken to the sun's lower limb.

Observation	Sun	Watch time	Vertical Angle	Horizontal angle from reference point to sun	
1st a. m.	o+	9h 44m 12s	34° 15' 00"	104° 8' 30"	to E.
3rd p. m.	+o	4 35 28		116 37 30	to W.
				12 29 00	Diff.
2nd a.m.	o+	9h 46m 28s	34° 40' 00"	104° 36' 30"	to E.
2nd p.m.	+o	4 33 10		117 5 30	to W.
		6 46 42		12 29 00	Diff.
3rd a.m.	o+	9h 49m 7s	35° 08' 00"	105° 10' 00"	to E.
1st p.m.	+o	4-40 36		117 38 30	to W.
				12 28 30	Diff.

One-half differences, or bearing angles from uncorrected north point to reference point:

By 1st obsn.	N. 6° 14' 30" E.
By 2nd obsn.	N. 6° 14' 30" E.
By 3rd obsn.	N. 6° 14' 15" E.
Mean	N. 6° 14' 25" E.
Differential azimuth correction	- 5' 08"
Mean true bearing of ref. pt.	N. 6° 9' 17" E.

½ dd :: ½ x 6.78 x 54.76 :: 186" log ½dd :: 2.269513
ø 38° 52'.4 N.; log cos ø 9.891278
½(t_1+t_2) 3h 23m 21s
~~50° 49' 15"~~ (50°50'15")
log sin ½ (t_1+t_2) 9.889400
9.780678 9.780678
log d Ad 2.488835

Differential azimuth correction 308"

5' 8"

ALTITUDE OBSERVATION OF THE SUN FOR AZIMUTH

(From the central pair of series)
(observed by equal altitude method)

Vertical angle to sun's lower limb	34° 40' 00"
Reduction to sun's center	+ 15 53
Refraction ((6000 ft.)	- 1 8
Parallax	+ 8
Sun's center h	34° 54' 53"

Sun's declination a. m. obsn. 7° 48'.8 N.
p. m. obsn. 7° 42'.5 N.

A. M. obsn.

Nat sin 7° 48'.8 .13594
" cos 38 52 .4 .77854
" cos 34 54 .9 .82001

Nat tan 38 52 .4 .80613
" tan 34 54 .9 .69800

$\frac{.13594}{.77854 \times .82001} - .80613 \times .69800$

+ .21294
- .56268

Cos A - .34974
A S. 69° 31' 42" E.

P. M. obsn.

Nat sin 7° 42'.5 .13413

$\frac{.13413}{.77854 \times .82001} - .80613 \times .69800$

+ .21010
- .56268

Cos A - .35258
A S. 69° 21' 18" W.

Deflection angle from reference point to NNE.:
104° 36'30" | 117° 5' 30"

Reduction to sun's center $\frac{15.9'}{\cos 34° 55'}$

- 19'24" | - 19' 24"

Horizontal angle to sun's center:

104° 17' 06" to E. | 116° 46' 06" to W.

Sun's azimuth as computed:
S.69° 31' 42" E. | S. 69° 21' 18" W.

True bearing of reference point:

N.173° 48' 48" E. | N. 186° 07' 24" W.
N. 6° 11' 12" E. | N. 6° 07' 24" E.

Mean true bearing of reference point:

N. 6° 09' 18" E.

P - 2

EQUAL ALTITUDE OBSERVATION OF THE SUN FOR AZIMUTH

Date: Sept. 22, 1944 | Observer: A. W. Brown | Instrument: Buff No 23815
Recorder: T. W. Crawford
Timer: T. W. Crawford

FINAL FIELD NOTES

Sept. 22, 1944, at the G.L.O. instrument testing station on the U.S. post-office roof, Denver, Colo., in latitude 39° 44' 56" N., longitude 104° 59' 22.6" W., with elevation approx. 5,200 ft. and temp. approx. 70° F., between 10h 24m a.m. and 3h 17m., p.m., m.w.t., I make a series of three equal altitude observations upon the sun for azimuth, reading the horizontal deflection angles from a point in the letter "R" of hotel sign "KENMARK", about 20 chs. to the SOUTH, SE. in a.m. to the sun's right limb and SW. in p.m. to the sun's left limb, equal vertical angles being taken to the sun's lower limb.

Observation	Sun	Watch time, m.r.t.	Vertical angle	Horizontal angle flag to sun.
1st a.m.	⊙	10h 24m 04s	37° 34' 00"	49° 51' 00" to SE.
3rd p.m.	⊙	3h 22m 27s	"	49° 42' 00" to SW.
				0° 09' 00" (Diff.)
2nd a.m.	⊙	10h 26m 50s	37° 59' 00"	49° 09' 00" to SE.
2nd p.m.	⊙	3h 19m 38s	"	48° 59' 00" to SW.
sum of hr. angles		4h 52m 48s		0° 10' 00" (Diff.)
mean hour angle		2h 26m 24s.		
3rd a.m.	⊙	10h 29m 42s	38° 24' 00"	48° 23' 00" to SE.
1st p.m.	⊙	3h 16m 49s	"	48° 14' 30" to SW.
				0° 08' 30" (Diff.)

One-half differences, or bearing angles for uncorrected south point of flag.

By 1st observation	0° 04' 30"
By 2nd observation	0° 05' 00"
By 3rd observation	0° 04' 15"
Mean	0° 04' 35"
Differential azimuth correction(–)	5' 12"
Mean true bearing of flag	S. 0° 0' 29" E.

FIELD RECORD

The hourly change in the sun's declination 0' 58.41" S.

The following computation is made to obtain the differential azimuth correction for the above series. The reduction is made by the following equation, where ½d and dA are each expressed in seconds:

$$dA\delta = \frac{\frac{1}{2}d\delta}{\cos\phi \ \sin\frac{1}{2}(t_1+t_2)}$$

$\frac{1}{2}d\delta = \frac{1}{2} \times 4.88 \times 58.41 = 143"$ log $\frac{1}{2}d\delta$ = 2.155336

ϕ = 39° 44' 56" N., log cos ϕ = 9.885844

$\frac{1}{2}(t_1+t_2)$ = 2h 26m 24s

= 36° 36' 00"; log sin $\frac{1}{2}(t_1+t_2)$ = 9.775410

9.661254 9.661254

log $dA\delta$ = 2.494082

$dA\delta$ = Differential azimuth correction = 312"

= 5' 12"

The following reduction to obtain the value of the differential azimuth correction is made with the use of Table 22 of the Standard Field Tables.

Latitude	$\frac{1}{2}(t_1+t_2)$ or hours from noon		
	2h	2h 26m 24s	3h
35°	2.44	2.05	1.73
39°44'56"		2.18	
40°	2.61	2.19	1.85

Declination coefficient 2.18

$d A \delta = 2.18 \times \frac{1}{2} d\delta = 2.18 \times 143" = 312"$

$dA\delta$ = differential azimuth correction = 5'12"

(This checks with the d.s.c. as computed by formula)

The 2nd a.m. and p.m. observations of the above series are selected for an example of the reduction to the sun's center and direct computation of the sun's azimuth, and true bearing of the flag by the equation:

$$\cos A = \frac{\sin}{\cos \phi \cos h} - \tan \phi \tan h$$

Vertical angle to sun's lower limb		37° 59' 00"
Reduction to sun's center	+	15' 58"
Parallax	+	7"
Refraction	−	1'00"
h: elev. of sun's center	=	38° 14' 05"

Declination of the sun, Gr. app. noon 0° 15' 44.8" N.

Difference in time to a.m. obsn.

For longitude 6h 59m 58s

For time, a.m. 2h 26m 24s

4.44h = 4h 26m 22s

Change in decl. to app. time of a.m. obsn.

4.44 × 58.41 = 259" = 4' 19" S.

Sun's decl., a.m. obsn. 0° 11' 26" N. 11' 26" N.

Diff. to p.m. obsn. already computed: (2 × 143) = 286" = 4m 46s S.

Sun's decl., p.m. obsn. 0° 6' 40" N.

		a.m. obsn.	p.m. obsn.
log cos ϕ = 9.885844		log sin δ = 7.521898 (+)	log sin δ = 7.287635 (+)
log cos h = 9.895137			
	9.780981	9.780981	9.780981
log tan ϕ = 9.919945		log 7.740917	log 7.506654
log tan h = 9.896474		nat. (+) .00551	nat (+) .00321
log	9.816419		
nat. (−)	.65527	(−) .65527	(−) .65527
Cos A =		(−) .64976	.65206
A = true bearing of sun =		S. 49° 28' 35" E.	S. 49° 18' 11" W.

Horizontal angles from flag to sun's right and left limbs:

	a.m.	p.m.
	49° 09' 00" to SE	48° 59' 00" to SW.
Reduction to sun's center $\frac{15.97}{\cos 38° 14' 05"}$ =	20' 20"	20' 20"
Hor. angle to sun's center	49° 29' 20"	49° 19' 20"
Sun's azimuth as computed above:	S. 49° 28' 35" E.	S. 49° 18' 11" W.
True bearing of flag	S. 0° 0' 45" E.	S. 0° 1' 9" E.

Mean true bearing of flag S. 0° 0' 57' E.

By above direct computation A: a.m., 49° 28' 35"

A: p.m. 49° 18' 11"

Difference 2dAδ = 10' 24"

dAδ = 5'12"

This value dAδ agrees with the same function as computed by the two previous methods.

Example of a stellar equal altitude observation for time, latitude, and azimuth.

Date: October 26, 1944. Observer: Ralph Gentry; Recorder: Elliot Bird; Time: Robert C. Yundt. Instrument: Gurley No. 38871.

At a transit point in Salt Lake City, Utah, in latitude 40° 44' 50" N., and longitude 111° 49' W., approximate elevation above sea level, 4900 feet; at $3^h\ 00^m$ p.m., app. t., I set the arcs of the solar unit, lat. 40° 44' 50" N., and decl. 12° 37' S., and orient to the meridian, and note that a beacon light, about 2 miles distant, bears N. 27° 06' E.

Star: 5/7 2.2
α Arietis
δ = + 23° 12.1'

Latitude: 40° 44' 50"

Gr. m.t. star's transit, Oct. 22:	$11^h\ 58^m$ p.m.
Red. to " 26:	- 15.7
" " long. 111° 49' W.	- 1.2
Star's transit, l.m.t. Oct. 26:	$11^h\ 41.1^m$ p.m.
Anticipated l.m.t. of obsn.	6 11 " "
Hour angle NE. (82° 30')	$5^h\ 30^m$

Watch fast of l.m.t., $0^h\ 27.2^m$

$$\sin h = \cos t \cos \phi \cos \delta + \sin \phi \sin \delta \qquad \cos A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

	cos	sin	
t =	.1305		
ϕ =	.7576	.6528	
δ =	.9191	.3939	
	.0909	.2571	(Products)
		.0909	
sin h =		.3480	(Sum)
h = 20° 22'			

	cos	sin	tan	
ϕ =	.7576		.8616	
h =	.9375		.3712	
δ =		.3939		
	.7103		- .3198	(Products)
		.5546	+ .5546	(Fraction)
cos A =			+ .2348	(Diff.)
A = N. 76° 25' E.				

With the above settings, at $6^h\ 11^m$ p.m., l.m.t., I find the star in good position and proceed with a series of four equal altitude observations.

Observation	Watch time	Vertical angle	Horizontal angle beacon light to star	
1st p.m., Oct. 26	$6^h\ 56^m\ 58^s$	23° 29' 30"	51° 46' 00"	to NE.
4th a.m., Oct. 27	5 21 32		105° 57' 30"	N. to NW.
Sum of hour angles	$10^h\ 24^m\ 34^s$		54° 11' 30"	(Diff.)
Mean hour angle	5 12 17			
Watch time, star's transit, Oct. 27	$0^h\ 09^m\ 15^s$			
2nd p.m., Oct. 26	$7^h\ 00^m\ 12^s$	24° 07' 30"	52° 16' 30"	to NE.
3rd a.m., Oct. 27	5 18 16		106 29 00	N. to NW.
Sum of hour angles	$10^h\ 18^m\ 04^s$		54° 12' 30"	(Diff.)
Mean hour angle	$5^h\ 09^m\ 02^s$			
Watch time, star's transit, Oct. 27	$0^h\ 09^m\ 14^s$			
3rd p.m., Oct. 26	$7^h\ 04^m\ 54^s$	25° 00' 30"	52° 58' 00"	to NE.
2nd a.m., Oct. 27	5 13 35		107 10 00	N. to NW.
Sum of hour angles	$10^h\ 08^m\ 41^s$		54° 12' 00"	(Diff.)
Mean hour angle	$5^h\ 04^m\ 20^s$			
Watch time, star's transit, Oct. 27	$0^h\ 09^m\ 15^s$			
4th p.m., Oct. 26	7 11 32	26° 15' 00"	53° 59' 00"	to NE.
1st a.m., Oct. 27	5 06 54		108 10 30	N. to NW.
Sum of hour angles	$9^h\ 55^m\ 22^s$		54° 11' 30"	(Diff.)
Mean hour angle	$4^h\ 57^m\ 41^s$			
Watch time, star's transit, Oct. 27	$0^h\ 09^m\ 13^s$			

One-half differences of horizontal angles, or bearings of reference point, and watch time of star's transit.

	Bearing of reference point	Watch time of star's transit
By 1st obsn.	= N. 27° 05' 45" E.	$0^h\ 09^m\ 15^s$ a.m.
" 2nd "	= N. 27 06 15 E.	0 09 14 " "
" 3rd "	= N. 27 06 00 E.	0 09 15 " "
" 4th "	= N. 27 05 45 E.	0 09 13 " "
Mean bearing of reference point	= N. 27° 05' 56" E.	

Mean watch t., star's transit, Oct.27	$0^h\ 09^m\ 14^s$ a.m.
Red. to Oct. 26	+12
Star's transit, p.m., l.m.t., Oct. 26	$11^h\ 41^m\ 06^s$
Watch fast of l.m.t.	$0^h\ 28^m\ 08^s$

Meridian altitude observation of the star for latitude.

Watch time of star's transit, Oct. 27, $0^h\ 09^m\ 14^s$ a.m.

Settings:		
	90°	00'
δ ≠	23	12
90°+ δ ≠	113°	12'
ϕ ≠	40	45
v ≠	72°	27' = (90° + δ) − ϕ

Telescope	Watch time	Vertical angle
Direct	$0^h\ 05^m\ 16^s$ a.m.	72° 28' 00"
Reversed	0 07 35 " "	72 27 00
Direct	0 11 22 " "	72 29 00
Reversed	0 14 42 " "	72 26 00
Mean	0h 09m ~~59s~~ 22 44s a.m.	72° 27' 30"
Refraction in zenith distance = 19"x.85 (coefficient for elevation of 4900 feet)		− 16"
		h = 72° 27' 14"
δ = 23° 12' 06"; 90° + δ		−113 12 06
Latitude = 40° 44' 52" N. = (90° + δ) − h		= 40° 44' 52"

Ralph Gentry

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Stellar Equal Altitude Observation for
Time, Azimuth and Latitude.

Date: November 8, 1944

Instrument: Buff No. 23829.

Observer: Quintin Campbell
Recorder: Roger F.Wilson
Time : Charles E.Hunter

Selected Star: 27/54 1.3
α Piscis Australis (Fomalhaut)
- 29° 55.05'

At observing station near the center of sec. 36, T. 1 N., R. 8 E., S. B. M., California, in latitude 34° 07.6' N., and longitude 116° 06' W., I make a series of six equal altitude observations on the star for time and azimuth, reading the horizontal deflection angle to the left and to the right of an illuminated flag, about 15 chs. S. and E. of my station. The star is also observed at its maximum elevation for vertical angle to obtain the latitude of the station.

Observation.	Watch time.	Vertical angle	Horizontal angle (Flag to star)	
1.	6h 05m 57s p.m.	23° 13'	13° 24' 00"	to SE.
12.	8 47 08		24° 37' 00"	to SW.
			11° 13' 00"	(Diff.)
2.	6h 10m 51s	23° 31'	12° 18' 30"	to SE.
11.	8 42 12		23° 31' 30"	to SW.
			11° 13' 00"	(Diff.)
3.	6h 15m 40s	23° 50'	11° 12' 30"	to SE.
10.	8 37 25		22° 25' 30"	to SW.
			11° 13' 00"	(Diff.)
4.	6h 20m 38s	24° 06' 30"	10° 05' 30"	to SE.
9.	8 32 28		21° 18' 30"	to SW.
			11° 13' 00"	(Diff.)
5.	6h 25m 20s	24° 22'	8° 59' 30"	to SE.
8.	8 27 42		20° 12' 30"	to SW.
			11° 13' 00"	(Diff.)
6.	6h 30m 23s	24° 26'	7° 49' 30"	to SE.
7.	8 22 41		19° 02' 30"	to SW.
			11° 13' 00	(Diff.)

One-half differences, or bearing angles from South point to flag.

By 1st obsn.	=	S. 5° 36' 30" E.	
" 2nd "	=	" "	
" 3rd "	=	" "	
" 4th "	=	" "	
" 5th "	=	" "	
" 6th "	=	" "	

Mean true bearing to flag - S. 5° 36' 30" E.

Time.

Mean of watch readings; obsns. 1 to 6	:	6h 18m 08s p.m.
Mean of watch readings; obsns. 7 to 12	:	8h 34m 56s
Sum	:	14h 52m 64s
½ sum; Watch time of transit of Star	:	7h 26m 32s p.m.

Greenwich mean time; star's transit Nov.1	:	8h 11m p.m.
Reduced to Nov. 8	:	– 27.5
Reduced to longitude 116° 06' W.	:	– 1.3
Star's transit, l.m.t.	:	7h 42.2m p.m.
Longitude correction to Pac.Standard Time	:	– 15.6
Star's transit, P. S. T.	:	7h 26.6m
Observed P. S. T. of Star's transit	:	7h 26.5m
Watch slow of Pacific Standard Time	:	0.1m

Latitude.

Observed altitude, direct			:	25° 58' 30"
" " , reversed			:	26° 00' 00"
Mean observed altitude	:	v	=	25° 59' 15"
Refraction	:	r	=	– 0° 1' 58"
True vertical angle	:	h	=	25° 57' 17"
Star's declination	:	δ	=	29° 55' 03" S.
Sum			=	55° 52' 20"
Latitude:		ϕ	=	90° – h ± δ

90° 00' 00"
– 55° 52' 20"

Reduced latitude : ϕ = 34° 07' 40" N.

Example of direct altitude observation of the sun for azimuth and time, sun south declination:

Date:	November 9, 1944.	Instrument:	Buff No. 14187.
Observer:	George W. Johnson	Recorder:	Ray W. Garrett.

Collimator test of vertical circle, Sept. 5, 1944:

Telescope direct, sights OK at zero		0° 00' 00"
Telescope reversed, sights high at zero		0 01 00
Mean index correction at zero		- 0' 30"
At true vertical angle	+40° 00' 00"	-40° 00' 00"
Mean corrected value	+40 00 30	-40 00 00
Mean index correction	- 0' 30"	0' 00"

Transcribed field notes.

Nov. 9, 1944, at my station on the random line between secs. 13 and 14, T. 1 S., R. 27 W., 5th Prin. Mer., Arkansas, in latitude 34° 39' 37" N., and longitude 93° 50' W., elevation above sea level approximately 1000 ft., and temperature approximately 40° F., at $8^h\ 56^m$ a.m. app. time, in order to verify the adjustments of the solar unit, I make a series of three altitude observations of the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun, and reading the horizontal angle from a flag on my retracement line set by solar transit orientation S. 0° 40' W., SE. to the sun. My watch carries approximate apparent time.

Observation.	Telescope.	Sun.	Watch time.	Observed vertical angle.	Horizontal angle, flag to sun.
1st	Direct		$8^h\ 56^m\ 00^s$	22° 43' 00"	49° 03' 00"
"	Reversed		8 57 00	22 18 00	48 15 00
	Mean		$8^h\ 56^m\ 30^s$	22° 30' 30"	48° 39' 00"
2nd	Reversed		$8^h\ 57^m\ 30^s$	22° 57' 00"	48° 42' 00"
"	Direct		8 58 30	22 34 00	47 57 00
	Mean		$8^h\ 58^m\ 00^s$	22° 45' 30"	48° 19' 30"
3rd	Direct		$8^h\ 59^m\ 00^s$	23° 15' 00"	48° 21' 00"
"	Reversed		9 01 00	22 50 00	47 33 00
	Mean		$9^h\ 00^m\ 00^s$	23° 02' 30"	47° 57' 00"

By 1st obsn. flag bears	S. 0° 39' 41" W.
" 2nd " " "	S. 0 39 53 W.
" 3rd " " "	S. 0 39 57 W.
Mean, true bearing of flag	S. 0° 39' 50" W.

Watch fast of l.m.t., by 2nd obsn. $15^m\ 57^s$

Field record.

The declination of the sun for the mean period of the three observations = 16° 56' 38" S.

The following reductions are made to obtain the true vertical angles of the above observations:

		1st obsn.	2nd obsn.	3rd obsn.
	v =	22° 30' 30"	22° 45' 30"	23° 02' 30"
Transit index correction ....,	=	- 0 30	- 0 30	- 0 30
Refraction (Coef. : .98 x 1.02 = 1.00)	=	- 2 19	- 2 17	- 2 15
Sun's parallax	=	+ 8	+ 8	+ 8
	h =	22° 27' 49"	22° 42' 51"	22° 59' 53"

The above observations are reduced for azimuth by the equation:

$$\operatorname{Cos} A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

The function "sin δ" becomes negative for south declinations.

1st obsn.:

$\phi = 34°\ 39'\ 37''$ $h = 22°\ 27'\ 49''$ $\delta = 16°\ 56'\ 38''$

cos ϕ = .822539	sin δ = -.291435	tan ϕ = .691408
cos h = .924122		tan h = .413470
.760126	.760126	-.285876
	-.383404	-.383404

Cos A = -.669280
A = S. 47° 59' 19" E.
Hor. ang. = 48 39 00

Bearing of flag = S. 0° 39' 41" W.

2nd obsn.:

$h = 22°\ 42'\ 51''$

cos ϕ = .822539	sin δ = -.291435	tan ϕ = .691408
cos h = .922443		tan h = .418600
.758745	.758745	-.289423
	-.384101	-.384101

Cos A = -.673524
A = S. 47° 39' 37" E.
Hor. ang. = 48 19 30

Bearing of flag = S. 0° 39' 53" W.

3rd obsn.:

$h = 22°\ 59'\ 53''$

cos ϕ = .822539	sin δ = -.291435	tan ϕ = .691408
cos h = .920518		tan h = .424435
.757162	.757162	-.293458
	-.384904	-.384904

Cos A = -.678362
A = S. 47° 17' 03" E.
Hor. ang. = 47 57 00

Bearing of flag = S. 0° 39' 57" W.

The second observation is reduced for time by the equation:

$$\operatorname{Cos} t = \frac{\sin h}{\cos \phi \cos} - \tan \phi \tan \delta$$

The product "tan ϕ tan δ" is additive for south declinations.

cos ϕ = .822539	sin h = .386134	tan ϕ = .691408
cos δ = .956591		tan δ = .304660
.786833	.786833	.201644
	.490745	.490745

Cos t = .701381
t = 45° 27' 41"
= 3h 1m 51s
App. time = 8h 58m 09s a.m.
Equation of time = -16 06

L.m.t. of obsn. = 8h 42m 03s a.m.
Watch time of obsn. = 8 58 00 a.m.

Watch fast of l.m.t. = 15m 57s

The first of the above series is selected for an example of reduction for azimuth by the equation:

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin S \sin (S - \text{codec.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

$90° - \phi = 90°$	$-$	34° 39' 37"	= 55° 20' 23"	= colat.
$90° - \delta = 90°$	$-$	(−16 56 38)	= 106 56 38	= codec.
$90° - h = 90$	$-$	22 27 49	= 67 32 11	= coalt.

2S	= 229° 49' 12"
S	= 114 54 36
Codec.	= 106 56 38
S − codec.	= 7° 57' 58"

log sin S		9.957593
log sin (S − codec.)		9.141724
	log	9.099317
log sin colat.	9.915156	
log sin coalt.	9.965729	
log	9.880885	9.880885
log cos² ½ A	=	9.218432
log cos ½ A	=	9.609216
½ A	=	66° 00' 21"
A	=	N. 132 00 42 E.
	=	S. 47° 59' 18" E.
Hor. ang.	=	48 39 00
Bearing of flag	=	S. 0° 39' 42" W.

The second of the above series is selected for an example of reduction for azimuth by the equation:

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\cos \frac{1}{2}(\zeta+\phi+\delta) \sin \frac{1}{2}(\zeta+\phi-\delta)}{\cos \frac{1}{2}(\zeta-\phi-\delta) \sin \frac{1}{2}(\zeta-\phi+\delta)}}$$

	90° 00' 00"
h =	22 42 51

ζ =	67° 17' 09"	ζ =	67° 17' 09"
ϕ =	34 39 37	ϕ =	34 39 37
$\zeta+\phi$ =	101° 56' 46"	$\zeta-\phi$ =	32° 37' 32"
δ =	16 56 38 (−)	δ =	16 56 38 (−)
$\zeta+\phi+\delta$ =	85° 00' 08"	$\zeta-\phi+\delta$ =	15° 40' 54"
$\frac{1}{2}(\zeta+\phi+\delta)$ =	42° 30' 04"	$\frac{1}{2}(\zeta-\phi+\delta)$ =	7° 50' 27"
$\zeta+\phi$ =	101° 56' 46"	$\zeta-\phi$ =	32° 37' 32"
δ =	16 56 38 (−)	δ =	16 56 38 (−)
$\zeta+\phi-\delta$ =	118° 53' 24"	$\zeta-\phi-\delta$ =	49° 34' 10"
$\frac{1}{2}(\zeta+\phi-\delta)$ =	59° 26' 42"	$\frac{1}{2}(\zeta-\phi-\delta)$ =	24° 47' 05"

log cos $\frac{1}{2}(\zeta+\phi+\delta)$		9.867623
log sin $\frac{1}{2}(\zeta+\phi-\delta)$		9.935075
		9.802698
log cos $\frac{1}{2}(\zeta-\phi-\delta)$	9.958033	
log sin $\frac{1}{2}(\zeta-\phi+\delta)$	9.134873	
	9.092906	9.092906
log tan² ½ A		0.709792
log tan ½ A		0.354896
½ A		66° 10' 12"
A		N. 132° 20' 24" E.
		S. 47° 39' 36" E.
Hor. angle		48 19 30
Bearing of flag		S. 0° 39' 54" W.

The second of the above series is likewise selected for an example of reduction for time by the equation:

$$\operatorname{Tan} \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(\zeta+\phi-\delta)\ \sin \frac{1}{2}(\zeta-\phi+\delta)}{\cos \frac{1}{2}(\zeta+\phi+\delta)\ \cos \frac{1}{2}(\zeta-\phi-\delta)}}$$

$\log \sin \frac{1}{2}(\zeta+\phi-\delta)$		9.935075
$\log \sin \frac{1}{2}(\zeta-\phi+\delta)$		9.134873
		9.069948
$\log \cos \frac{1}{2}(\zeta+\phi+\delta)$	9.867623	
$\log \cos \frac{1}{2}(\zeta-\phi-\delta)$	9.958033	
	9.825656	9.825656

$\log \tan^2 \frac{1}{2} t = 9.244292$
$\log \tan \frac{1}{2} t = 9.622146$
$\frac{1}{2} t = 22° 43' 50"$.
$t = 45° 27' 40"$
$= 3^h 01^m 51^s$

App. time of observation	=	$8^h 58^m 09^s$ a.m.
Equation of time		-16 06
L.m.t. of observation	=	$8^h 42^m 03^s$ a.m.
Watch time of observation	=	8 58 00 a.m.
Watch fast of l.m.t.	=	$15^m 57^s$

The second of the above series is likewise selected for an example of reduction for time by the equation:

$$\operatorname{Sin} t = \frac{\sin A \cos h}{\cos \delta}$$

$A = 47° 39' 37"$ $h = 22° 42' 51"$ $= 16° 56' 38"$

$\log \sin A = 9.868741$
$\log \cos h = 9.964939$

9.833680
$\log \cos \delta = 9.980726$

$\log \sin t = 9.852954$
$t = 45° 27' 41" = 3^h 01^m 51^s$

Apparent time of observation	=	$8^h 58^m 09^s$ a.m.
Equation of time	=	-16 06
Local mean time of observation	=	$8^h 42^m 03^s$ a.m.
Watch time of observation	=	8 58 00 a.m.
Watch fast of local mean time	=	$15^m 57^s$

Note: This equation is not recommended for hour angles approaching 90° or 6 hours, as the sine function changes very slowly for angles approaching 90°.

Date:
October 21, 1944.
Time: l.m.t.

Observer:
E. D. Calvin

Instrument:
Buff No. 19422.

Example of direct altitude observation of the sun for azimuth and time, sun south declination:

Transcribed field notes.

October 21, 1944, at a transit point in Boise, Idaho, in latitude 43° 37' 13" N., and longitude 116° 12' W., altitude above sea level approximately 2700 ft., and temperature approximately 50° F., at 8^h 27^m a.m., app. time, I make a series of three altitude observations of the sun for azimuth, each with the telescope in direct and reversed positions, observing opposite limbs of the sun, and reading the horizontal angle from depot cupola, designated as flag, about 60 chs. to the SW., to the left to the sun. My watch carries approximate local mean time.

Observation.	Telescope.	Sun.	Watch time.	Observed vertical angle.	Horizontal angle, flag to sun.
1st	Direct		8^h 11^m 15^s	17° 29' 00"	88° 28' 00"
"	Reversed		8 12 19	17 08 30	88 46 00
	Mean		8^h 11^m 47^s	17° 18' 45"	88° 37' 00"
2nd	Direct		8^h 13^m 07^s	17° 48' 00"	88° 03' 00"
"	Reversed		8 14 29	17 28 30	88 16 00
	Mean		8^h 13^m 48^s	17° 38' 15"	88° 09' 30"
3rd	Direct		8^h 15^m 40^s	18° 13' 00"	87° 27' 00"
"	Reversed		8 16 50	17 53 30	87 43 00
	Mean		8^h 16^m 15^s	18° 03' 15"	87° 35' 00"

By 1st obsn. flag bears	S. 33° 12' 06" W.
By 2nd obsn. flag bears	S. 33 11 32 W.
By 3rd obsn. flag bears	S. 33 12 01 W.
Mean, true bearing of flag	S. 33° 11' 53" W.

Watch slow of l.m.t., 2nd obsn. = 22ˢ

Field record.

The declination of the sun for the mean period of the three observations is 10° 48' 49" S.

The following reductions are made to obtain the true vertical angles of the above observations:

	1st obsn.	2nd obsn.	3rd obsn.
v	17° 18' 45"	17° 38' 15"	18° 03' 15"
Refraction (Coef. = .92)	- 2 48	- 2 46	- 2 42
Parallax	+ 8	+ 8	+ 8
h	17° 16' 05"	17° 35' 37"	18° 00' 41"

The above observations are reduced for azimuth by the equation:

$$\text{Cos } A = \frac{\sin \delta}{\cos \varphi \cos h} - \tan \varphi \tan h$$

The function "sin δ" becomes negative for south declinations.

1st obsn.:

log cos φ	9.859695	log sin δ	9.273267	log tan φ	9.979076
" " h	9.979970			" " h	9.492556
log	9.839665		9.839665	log	9.471632
				nat −	.296232
		log	9.433602		
		nat	.271395	−	.271395
				Cos A −	.567627

True bearing of sun, A	S. 55° 24' 54" E.
Hor. ang., flag to sun	88 37 00
True bearing of flag	S. 33° 12' 06" W.

2nd obsn.:

log cos φ	9.859695	log sin δ	9.273267	log tan φ	9.979076
" " h	9.979195			" " h	9.501191
log	9.838890		9.838890	log	9.480267
				nat −	.302181
		log	9.434377		
		nat −	.271880	−	.271880
				Cos A −	.574061

True bearing of sun, A	S. 54° 57' 58" E.
Hor. ang., flag to sun	88 09 30
True bearing of flag	S. 33° 11' 32" W.

3rd obsn.:

log cos φ	9.859695	log sin δ	9.273267	log tan φ	9.979076
" " h	9.978178			" " h	9.512070
log	9.837873		9.837873	log	9.491146
				nat −	.309846
		log	9.435394		
		nat −	.272517	−	.272517
				Cos A −	.582362

True bearing of sun, A	S. 54° 22' 59" E.
Hor. ang., flag to sun	87 35 00
True bearing of flag	S. 33° 12' 01" W.

The second observation is reduced for time by the equation:

$$\text{Cos } t = \frac{\sin h}{\cos \varphi \cos \delta} - \tan \varphi \tan \delta$$

The product "tan φ tan δ" is additive for south declinations.

log cos φ	9.859695
" " δ	9.992219
log	9.851914

log tan φ	9.979076
" " δ	9.281048
log	9.260124
nat +	.182022

log sin h	= 9.480386
log (cos φ cos δ)	= 9.851914
log	= 9.628472
nat +	= .425081
tan φ tan δ +	= .182022
cos t	= .607103

t = 52° 37' 11" = $3^h\ 30^m\ 29^s$	= $8^h\ 29^m\ 31^s$ a.m.
Equation of time	= - 15 21
L.m.t. of observation	= $8^h\ 14^m\ 10^s$ a.m.
Watch time of observation	= 8 13 48 a.m.
Watch slow of l.m.t.	= 22^s

The first of the above series is selected for an example of reduction by the equation:

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin S \sin (S\text{-codec.})}{\sin \text{colat.} \sin \text{coalt.}}}$$

90°-φ = 90° - 43°37'13"	=	46°22'47"	= colat.
90°-δ = 90°-(-10°48'49")	=	100°48'49"	= codec.
90°-h = 90° - 17°16'05"	=	72°43'55"	= coalt.
2S	=	219°55'31"	
S	=	109°57'45"	
codec.	=	100°48'49"	
S-codec.	=	9°08'56"	

log sin S		9.973089
log sin (S-codec.)		9.201399
		log 9.174488
log sin colat.	9.859695	
log sin coalt.	9.979970	
log	9.839665	9.839665
$\log \cos^2 \frac{1}{2} A$		9.334823
$\log \cos \frac{1}{2} A$		9.667412
$\frac{1}{2} A$		62°17'33"
A		N. 124°35'06" E.
True bearing of sun		S. 55°24'54" E.
Hor. ang. flag to sun		88°37'00"
True bearing of flag		S. 33°12'06" W.

The second of the above series is selected for an example of reduction by the equation:

$$\operatorname{Tan} \tfrac{1}{2} A = \sqrt{\frac{\cos \frac{1}{2}(\zeta+\varphi+\delta)\ \sin \frac{1}{2}(\zeta+\varphi-\delta)}{\cos \frac{1}{2}(\zeta-\varphi-\delta)\ \sin \frac{1}{2}(\zeta-\varphi+\delta)}}$$

$$
\begin{array}{rl}
 & 90°00'00" \\
h = & \underline{17°35'37"} \\
\zeta = & 72°24'23" \\
\varphi = & \underline{43\ 37\ 13} \\
\zeta+\varphi = & 116°01'36" \\
\delta = & \underline{10\ 48\ 49}\ \ (-) \\
\zeta+\varphi+\delta = & 105°12'47" \\
\frac{1}{2}(\zeta+\varphi+\delta) = & \underline{52\ 36\ 23} \\
\zeta+\varphi = & 116°01'36" \\
\delta = & \underline{10\ 48\ 49}\ \ (-) \\
\zeta+\varphi-\delta = & 126°50'25" \\
\frac{1}{2}(\zeta+\varphi-\delta) = & \underline{63\ 25\ 12}
\end{array}
\qquad
\begin{array}{rl}
\zeta = & 72°24'23" \\
\varphi = & \underline{43\ 37\ 13} \\
\zeta-\varphi = & 28°47'10" \\
\delta = & \underline{10\ 48\ 49}\ \ (-) \\
\zeta-\varphi+\delta = & 17°58'21" \\
\frac{1}{2}(\zeta-\varphi+\delta) = & \underline{8\ 59\ 10} \\
\zeta-\varphi = & 28°47'10" \\
\delta = & \underline{10\ 48\ 49}\ \ (-) \\
\zeta-\varphi-\delta = & 39°35'59" \\
\frac{1}{2}(\zeta-\varphi-\delta) = & \underline{19\ 47\ 59}
\end{array}
$$

log cos $\frac{1}{2}(\zeta+\varphi+\delta)$		9.783394
log sin $\frac{1}{2}(\zeta-\varphi+\delta)$		9.951488
		9.734882
log cos $\frac{1}{2}(\zeta-\varphi-\delta)$	9.973535	
log sin $\frac{1}{2}(\zeta-\varphi+\delta)$	9.193667	
	9.167202	9.167202
	log $\tan^2 \frac{1}{2}$ A	0.567680
	log tan $\frac{1}{2}$ A	0.283840
	$\frac{1}{2}$ A	62°31'01"
	A	N. 125°02'02" E.
	True bearing of sun	S. 54°57'58" E.
	Hor.ang., flag to sun	88°09'30"
	True bearing of flag	**S. 33° 11' 32" W.**

The second of the above series is likewise selected for an example for the computation for time by the equation:

$$\operatorname{Tan} \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(\zeta+\varphi-\delta)\ \sin \frac{1}{2}(\zeta-\varphi+\delta)}{\cos \frac{1}{2}(\zeta+\varphi+\delta\ \ \cos \frac{1}{2}(\zeta-\varphi-\delta)}}$$

log sin $\frac{1}{2}(\zeta+\varphi-\delta)$		9.951488
log sin $\frac{1}{2}(\zeta-\varphi+\delta)$		9.193667
		9.145155
log cos $\frac{1}{2}(\zeta+\varphi+\delta)$	9.783394	
log cos $\frac{1}{2}(\zeta-\varphi-\delta)$	9.973535	
	9.767929	9.756929
log $\tan^2 \frac{1}{2}$ t		9.388226
log tan $\frac{1}{2}$ t		9.694113
$\frac{1}{2}$ t	26°18'35"	
t	52°37'10"	= $3^h30^m29^s$

Apparent time of observation	=	$8^h\ 29^m\ 31^s$ a.m.
Equation of time	=	-15 21
L.m.t. of observation	=	$8^h\ 14^m\ 10^s$ a.m.
Watch time of observation	=	8 13 48 a.m.
Watch slow of local mean time	=	22^s

FOR TEST OF THE SOLAR UNIT

Basswood Lake, Minn. Latitude: 48°03'30"N. Longitude: 91°30'35"W.

July 31, 1945. Arthur D. Kidder, District Cadastral Engineer, observing and recording; watch reading approximate local mean time. Buff No. 19421 General Land Office solar transit.

July 31, 1945, at the camp site in sec. 33, T. 65 N., R. 9 W., 4th Prin. Mer., Minnesota, on the south shore of Basswood Lake in the International Boundary Waters, in latitude 48°03'30"N., and longitude 91°30'35"W., I examine the adjustments of General Land Office solar transit, Buff No. 19421, and find that no corrections are required.

July 31: I make an altitude observation on the sun, p.m., for time and azimuth; and a sunset hour angle observation on Polaris for latitude and azimuth.
Aug. 1: at apparent noon, a meridian observation on the sun for time and latitude.

July 31: At 3^h54^m p.m., app.t., I set 48°03'30"N. on the lat. arc; 18°13'N. on the decl. arc; and orient to the meridian by observation with the solar unit. I find a suitable azimuth mark on the center of the west chimney of the main cabin of the Canadian Ranger Station, which bears N.37°03'30"E., about 4 Mi. dist.

July 31: Altitude observation on the sun:

Tel.	Sun	Watch Time p.m.	Hor. Ang. NE-N-W-SW		Observed Vert. Ang.
Dir.		$4^h\ 22^m\ 30^s$	133°35'00"		30°31'30"
Rev.		4 27 30	133°12'00"		30 13 30
Mean		$4^h\ 25^m\ 00^s$	133°23'30"	v =	30°22'30"
				r =	- 1 39
				p =	+ 08
			Decl. = 18°11'30"N.	h =	30°21'

83°40'00" = A = Reduced azimuth of sun.

		217°03'30"	Reduced app.t. of obsn. =	$4^h\ 18^m\ 08^s$
L.M.T.	$4^h\ 24^m\ 22^s$	- 180°00'00"	Equation of time =	+ 6 14
Watch fast	$0^m\ 38^s$	N.37°03'30"E. = Indicated bearing of reference mark.		

July 31: Polaris at sunset:

Tel.	Watch Time p.m.	Hor.Ang. NE-N			Observed Vert.Ang.
			Gr. U.C. Aug. 1 =	$5^h08.3^m$ a.m.	
			Red. for long.	- 1.0	
Dir.	$7^h\ 08^m\ 27^s$	36°19'00"	U.C., l.m.t. =	$5^h07.3^m$ a.m.	47°11'00"
Rev.	7 14 39	36 17 30			47 12 00
Mean	$7^h\ 11^m\ 33^s$	36°18'15"	Decl. = 88°59'59"N.	v =	47°11'30"
				r =	- 0 56
Watch fast of l.m.t.	$0^m\ 38^s$			h =	47°10'34"
			Vert. Ang. correction to elev. of pole, add		0°51'36"
L.M.T.	$7^h\ 10^m\ 55^s$		Latitude	=	48°02'10"
U.C., Aug. 1 a.m.	12 $5^h\ 07^m\ 18^s$				
Hour Ang. east	$9^h\ 56^m\ 23^s$				

N.00°44'48"E. = A = Reduced azimuth of Polaris

N.37°03'03"E. = Indicated bearing of reference mark.

N.37°03'30"E. = Same, by alt. obsn. on the sun.

N.37°03'16"E. = Mean.

Aug. 1: The sun at meridian passage:

Tel.	Sun	Watch Time			Observed Vert. Ang.
Rev.		$12^h\ 05^m\ 43^s$ $1\ 06$	Add to sun's center.		59°40'30"
		$12^h\ 06^m\ 49^s$	$12^h\ 00^m\ 00^s$ App. noon $+6\ 11$ Equation of time.,		
Dir.					60°11'30"
L.M.T.		$12^h\ 06^m\ 11^s$		Mean, v =	59°56'00"
				r =	- 33
Watch fast		$0^m\ 38^s$		p =	+03
Ditto, the sun, p.m., July 31		$0^m\ 38^s$	90°00'00" Decl. = 17°59'20"N.	h =	59°55'30" 107 59 20

Latitude	= 48°03'50"
By Polaris	= 48 02 10
Mean	= 48°03'00"

Reduction of the altitude observation on the sun, July 31:

φ = 48°03'30"N. δ = 18°11'30"N. h = 30°21'00"

$$\cos t = \frac{\sin h}{\cos\varphi\cos\delta} - \tan\varphi\tan\delta \qquad \cos A = \frac{\sin\delta}{\cos\varphi\cos h} - \tan\varphi\tan h$$

log	cos	sin	tan	cos	sin	tan
h =		9.703533		9.935988		9.767545
φ =	9.825020		10.046452	9.825020		10.046452
δ =	9.977732		9.516697		9.494428	
	9.802752	9.802752	9.563149	9.761008	9.761008	9.813997
		9.900781			9.733420	
nat		.79576 .36572	.36572		.54128	.65162 .54128
	cos t =	.43004 (+)		cos A =		.11034 (-)

t = 64°32'   A = S.83°40'00"W.

= $4^h 18^m 08^s$ = App. time of obsn.
$+ 6\ 14$ = Equation of time
$4^h 24^m 22^s$ = Local mean time.

Interpolation from tables in the Ephemeris: Polaris obsn.: July 31:

Hour angle.	For azimuth. Latitude. 48°	48°03'30"	50°	For latitude. Declination. 88°59'50"	88°59'59"	89°00'10"
$9^h 46.4^m$	'	'	'	0°50'37"		0°50'20"
48.4	47.1		49.0			
56.4	44.55	44.6	46.3	0 51 43	+0 51 35	0 51 26
58.4	43.9		45.6	0 51 57		0 51 40

For δ = 88°59'59" +0.2

A = N.0°44'48"E.

+ 01"	Supplemental correction.
+0°51'36"	Vert. Ang. correction to elev. of pole.

On Aug. 1 and 2, at about half hour intervals from 8:30 a.m. to 4:45 p.m., I make orientation tests of Buff solar transit No. 19421, including latitude tests by noon observation both days. The tests give a maximum orientation to the right 0'00"; maximum to the left 1'00" (N.0°01'00"W.); the mean of all a.m. tests, 0'25" to the left; the mean of all p.m. tests, 0'45" to the left; the mean of all, a.m. and p.m., 0'38" to the left (N.0°00'38"W.). The tests indicate that a slight improvement can be made to correct the vertical plane of the solar telescope for true parallel with the vertical plane of the transit. The difference is slight, being well within the limit of tolerance (1'30") that is permitted; I defer disturbing the adjustments pending more extended performance tests. The latter, as made by the chief of field party at frequent intervals during the period of the survey, showed normal satisfactory orientation without changing the adjustments.

CLOSING MEMORANDUM

The methods long in use have been developed for the correct operation of the General Land Office solar transit, in which the solar unit is depended upon for a large share of the line running, in combination with observations on the sun, Polaris, and the brighter stars for the more exact determination of time, latitude, and azimuth, where needed. The performance that is required of the solar transit is for uniform orientation, within the usual hours of observation, holding to within 1'30" of the true meridian. The field tests are carried along continually. This is done by comparing the orientation of the solar unit with a carefully determined azimuth line that is based upon a Polaris observation, or by altitude observation on the sun. Various combinations of these methods are employed, and frequently coupled with one or more observations upon a bright star, or stars, within the equatorial belt.

It will be noted that in most of the examples the values in the reductions have been carried to seconds of arc; this is done in order that the final result in azimuth may be reported ordinarily to the nearest even minute. Values in azimuth to the nearest even 30", 15", or 10", are not given in the transcribed field notes unless the nature of the survey and the basic observations justify that refinement.

The field calculations for latitude are always carried to seconds so that successive observations may be compared, usually all reduced to the south boundary of the township, and to have at hand a value to the nearest 30" for setting the latitude arc of the solar unit.

INT.-DUP. SEC., WASH., D.C. 23480

M E M O R A N D U M

The arrangement of the astronomical data in the Ephemeris[1] that is published annually by the Bureau of Land Management conforms to the methods and examples for the determination of time, latitude, and azimuth, as practiced by the cadastral surveyors; these are set out and explained in the Manual of Instructions for the Survey of the Public Lands of the United States, edition of 1947.

The methods long in use have been developed for the best operation of the General Land Office solar transit in combination with observations on the sun, Polaris, and the brighter equatorial stars for the determination and verification of time, latitude, and azimuth. In this practice the data for the sun are required in terms of the daily apparent positions for the Greenwich meridian; for all stellar data in terms of the Greenwich meridian, mean time, and mean time intervals.

The necessary mathematical tables are carried in the Standard Field Tables[1] published by the Bureau of Land Management.

The stellar numbers, as 4/6 for Polaris, refer to No. 4 in the list of 28 stars in the Ephemeris of the Bureau of Land Management, and No. 6 in the list of 55 stars published in the American Nautical Almanac.

Key to Symbols

$\approx$	Approximation	v	= Observed vertical angle
ϕ	Latitude	r	= Refraction in zenith distance
δ	Declination	p	= Parallax of the sun
ζ	Zenith distance	h	= Vertical angle corrected for refraction and parallax
A	= Horizontal angle counting from the meridian.	t	= Hour angle, mean time or sidereal, as required for the equation.

Footnote
[1] Superintendent of Documents, Government Printing Office, Washington 25, D.C.